ADITYA ENGINEERING COLLEGE (A)

UNIT - IV

Impact of Jet on Vanes
&
Introduction to Hydraulic Machinery

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IMPACT OF JET ON VANES

- Hydrodynamic force on jets on stationary and moving flat, Inclined and curved vanes,
- Jet striking centrally and at tip,
- Velocity diagrams, Work done and Efficiency,
- Flow over Radial vanes

Describe the concept of Impact of jet on plate in various condition

- The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid if flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate.
- This force is obtained from Newton’s second law of motion or from impulse-moment equation.

Impact of Jets

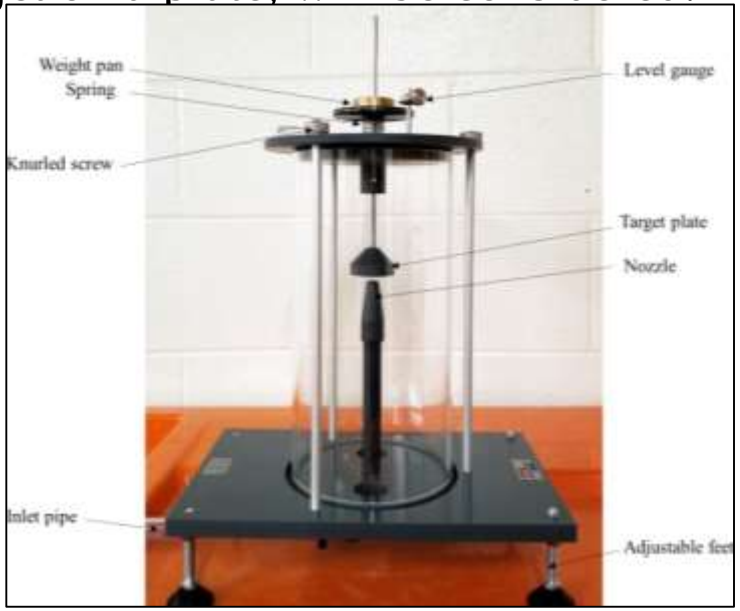
- The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the wheel
- Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving. In this chapter, the following cases of the impact jet i.e., the force exerted by the jet on a plate, will be considered.

1. Force exerted by a jet on a stationary plate when

- Plate is vertical to the jet,
- Plate is inclined to the jet, and
- Plate is curved.

2. Force is exerted by the jet on the moving plate, when

- Plate is vertical to the jet,
- Plate is inclined to the jet, and
- Plate is curved

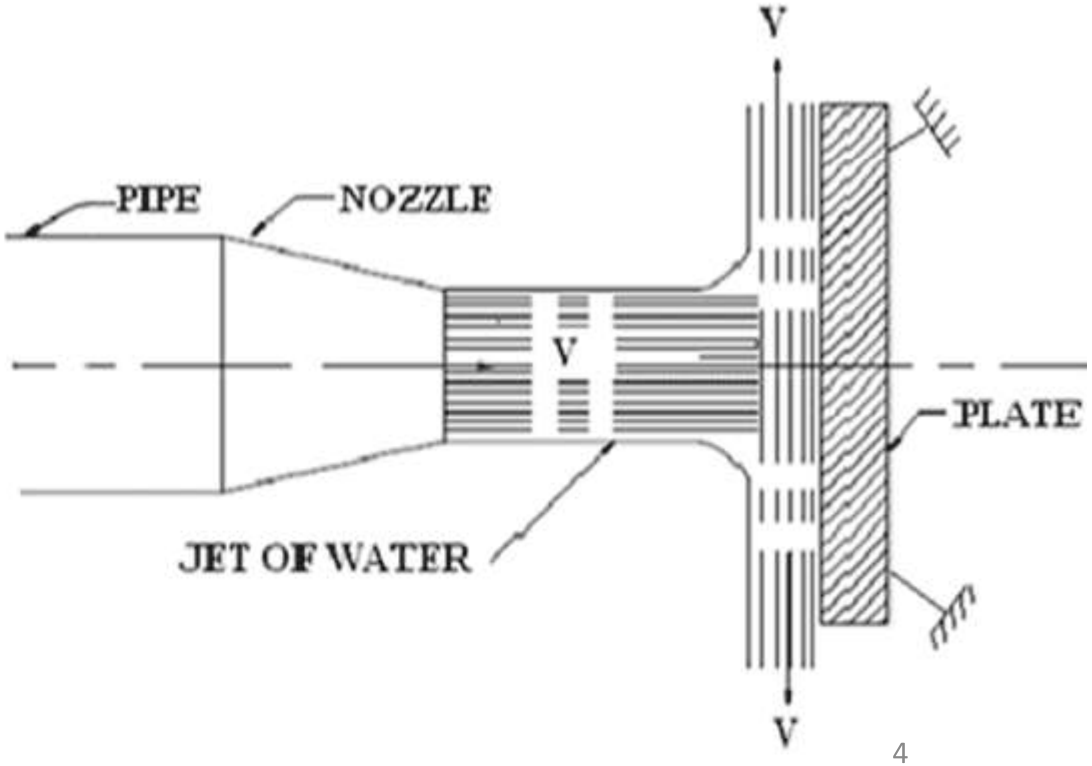


IMPULSE-MOMENTUM PRINCIPLE

- The impulse-momentum theorem states that the change in momentum of an object equals the impulse applied to it. The impulse-momentum theorem is logically equivalent to Newton's second law of motion (the force law).
- From Newton's 2nd Law: $F = m\,a = m\,(V_1 - V_2) / t$
- **Impulse** of a force is given by the change in momentum caused by the force on the body.
  $Ft = mV_1 - mV_2$ = Initial Momentum – Final Momentum.

1. A. Force exerted by a jet on a stationary vertical plate

- Consider a jet of water coming out of the nozzle, strikes a flat vertical plate as shown in the Figure.
- The jet after striking the plate will move along the plate. But the plate is right angles to the jet. Hence the jet after striking will get deflected by 90°. Hence the component of the velocity of the jet, in the direction of the jet, after striking will be zero. The force exerted by the jet on the plate in the direction of the jet.
- Fx = Rate of change of momentum in the direction of force
  = (initial momentum - final momentum) / time



$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

= (Mass/sec) × (velocity of jet before striking – velocity of jet after striking)

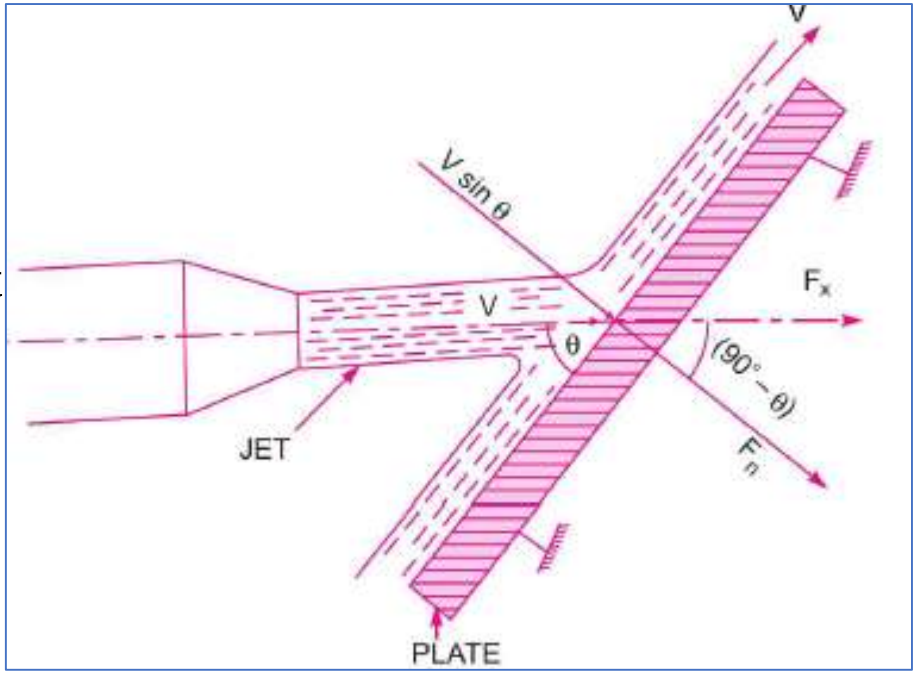
$$= \rho a V[V - 0] \qquad (\because \quad \text{mass/sec} = \rho \times a\, V)$$

$$= \rho a V^2 \qquad \ldots(17.1)$$

- For deriving the above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity.
- If the force exerted **on the** jet is to be calculated then final minus the initial velocity is taken. But if the force exerted **by the** jet on the plate is to calculated, then initial velocity minus the final velocity is taken.

B. Force exerted by a jet on a stationary inclined flat plate

- Let a jet of water, coming out from the nozzle; strike an inclined flat plate as shown in the figure.



Let V = Velocity of jet in the direction of x,

θ = Angle between the jet and plate,

a = Area of cross-section of the jet.

Then mass of water per sec striking the plate = $\rho \times aV$.

- If the plate is assumed smooth and if it is assumed that there is no loss of energy due to the impact of the jet, then the jet will move over the plate after striking with a velocity equal to initial velocity i.e., with a velocity V.
- Let find the force exerted by the jet on the plate in the direction normal to the plate.
- Let this force is represented by F_n then, F_n = Mass of the jet striking per second * [initial velocity of the jet before striking in the direction of n - final velocity of the jet after striking in the direction of n].

$$= \rho aV [V \sin \theta - 0] = \rho aV^2 \sin \theta$$

This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow. Then we have,

F_x = component of F_n in the direction of flow

$$= F_n \cos(90° - \theta) = F_n \sin \theta = \rho A V^2 \sin \theta \times \sin \theta \quad (\therefore F_n = \rho aV^2 \sin \theta)$$

$$= \rho AV^2 \sin^2 \theta \quad ...(17.3)$$

F_y = component of F_n, perpendicular to flow

$$= F_n \sin(90° - \theta) = F_n \cos \theta = \rho AV^2 \sin \theta \cos \theta. \quad ...(17.4)$$

C. Force Exerted by A Jet on Stationary Curved Plate

We will see here three conditions as mentioned here.

1. Jet strikes the curved plate at the center

2. Jet strikes the curved plate at one end tangentially when the plate is symmetrical

3. Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical

ege (A)

(A) Jet strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 17.3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet = $-V\cos\theta$.

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

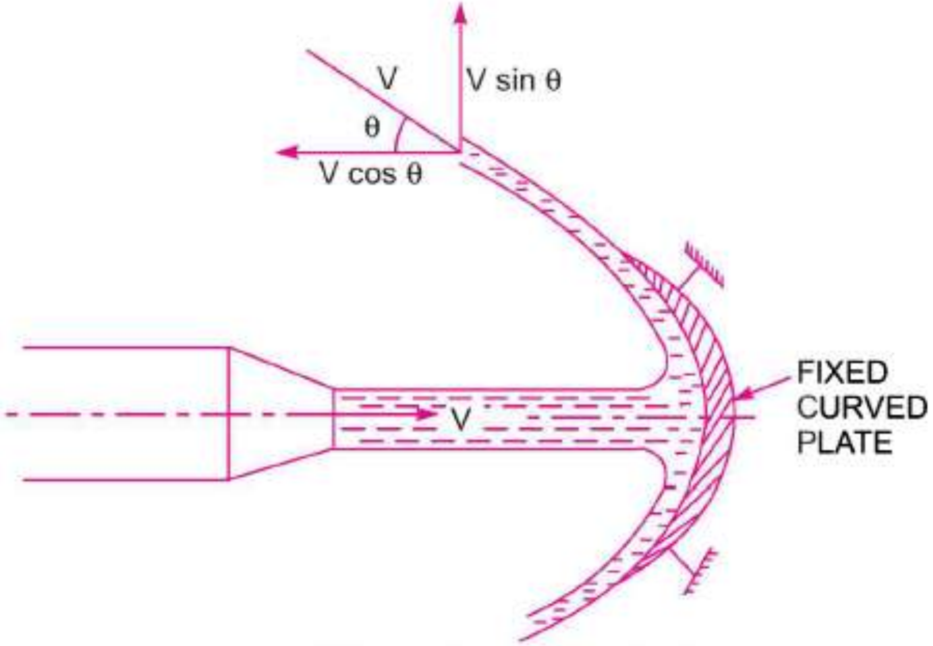
Component of velocity perpendicular to the jet = $V\sin\theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $-V\cos\theta$



$\therefore$ $$F_x = \rho aV[V - (-V\cos\theta)] = \rho aV[V + V\cos\theta]$$
$$= \rho aV^2[1 + \cos\theta]$$

Similarly, $$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$\therefore$ $$F_y = \rho aV[0 - V\sin\theta] = -\rho aV^2\sin\theta \quad \ldots(17.6)$$

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$ ...[17.6 (A)]

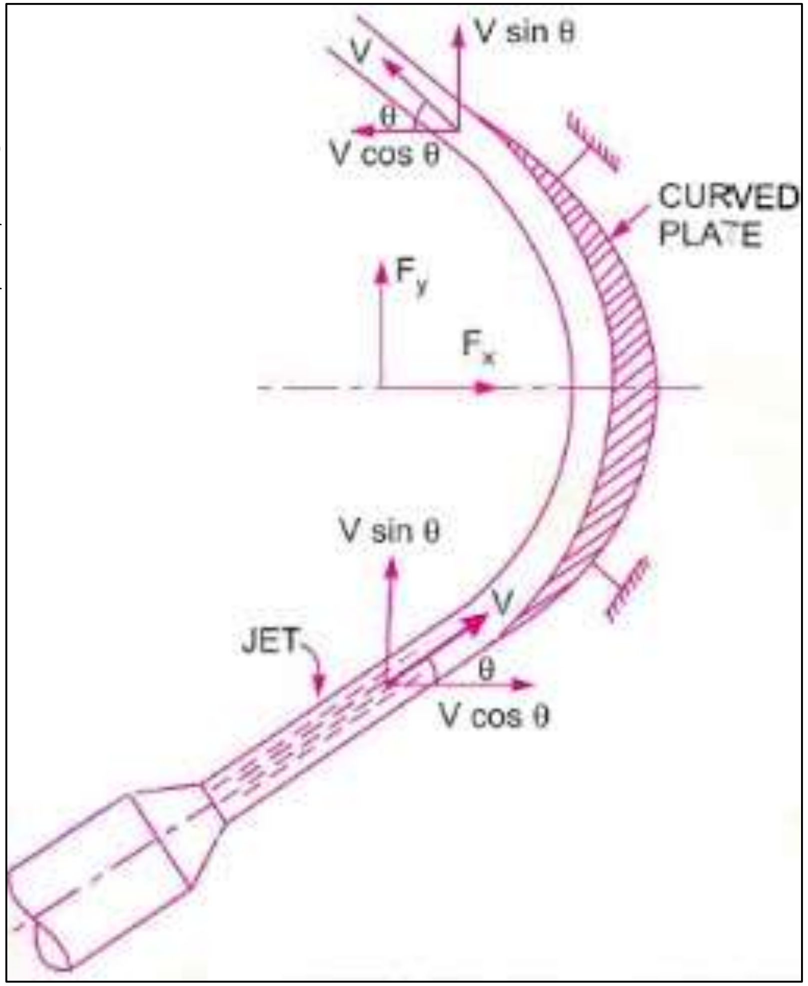
Jet strikes the curved plate at one end tangentially when the plate is Symmetrical:

Let us consider that a jet of water strikes a fixed stationary curved plate at its one end tangentially, when the plate is symmetrical, as displayed here in following figure.

Let V = Velocity of jet

θ = Angle made by jet with x - axis at inlet tip of the curved plate

Let us assume that the plate is smooth and there is no loss of energy due to impact of water jet. Water jet, after striking the stationary curved plate, will come at the outlet tip of the curved plate with similar velocity i.e. V in a direction tangentially to the curved plate.



Force exerted by the water jet in the x - direction

F_X = Mass per second x $[V_{1x} - V_{2x}]$,

Where, V_{1x} = Initial velocity in the x direction = $V \cos \theta$

V_{2x} = Final velocity in the x direction = - $V \cos \theta$

$F_X = \rho a V [V \cos \theta + V \cos \theta]$

$\boldsymbol{F_X = 2 \rho a V^2 \cos \theta}$

Force exerted by the water jet in the Y- direction

F_Y = Mass per second x $[V_{1y} - V_{2y}]$, Where,

V_{1y} = Initial velocity in Y direction = $V \sin \theta$

V_{2y} = Final velocity in Y direction = $V \sin \theta$

$\boldsymbol{F_Y = \rho a V [V \sin \theta - V \sin \theta]}$

Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical

When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis,

ϕ = angle made by tangent at outlet tip with x-axis.

The two components of the velocity at inlet are

$$V_{1x} = V\cos\theta \text{ and } V_{1y} = V\sin\theta$$

The two components of the velocity at outlet are

$$V_{2x} = -V\cos\phi \text{ and } V_{2y} = V\sin\phi$$

∴ The forces exerted by the jet of water in the directions of x and y are

$$F_x = \rho aV[V_{1x} - V_{2x}] = \rho aV[V\cos\theta - (-V\cos\phi)]$$

$$= \rho aV[V\cos\theta + V\cos\phi] = \rho aV^2[\cos\theta + \cos\phi] \quad \text{...(17.8)}$$

$$F_y = \rho aV[V_{1y} - V_{2y}] = \rho aV[V\sin\theta - V\sin\phi]$$

$$= \rho aV^2[\sin\theta - \sin\phi]. \quad \text{...(17.9)}$$

Problem 17.1 *Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20$ m/s.

The force exerted by the jet of water on a stationary vertical plate is given by equation (17.1) as

$$F = \rho aV^2 \quad \text{where } \rho = 1000 \text{ kg/m}^3$$

∴ $F = 1000 \times .004417 \times 20^2 \text{ N} = $ **1766.8 N. Ans.**

Problem 17.2 *Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.*

Solution. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100 \text{ m}$

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

But $$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08$ m/s.

Force on a fixed vertical plate is given by equation (17.1) as

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \quad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$= 13907.2$ N $=$ **13.9 kN. Ans.**

Problem 17.3 *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25 \text{ m/s}$.

Angle between jet and plate $\theta = 60°$

(*i*) The force exerted by the jet of water in the direction normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = \mathbf{2390.7 \text{ N. Ans.}}$$

(*ii*) The force in the direction of the jet is given by equation (17.3),

$$F_x = \rho a V^2 \sin^2 \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin^2 60° = \mathbf{2070.4 \text{ N. Ans.}}$$

Problem 17.4 *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Solution. Given :

Diameter of jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4} (.05)^2 = .001963 \text{ m}^2$

Angle, $\theta = 30°$

Force in the direction of jet, $F_x = 1471.5 \text{ N}$

Force in the direction of jet is given by equation (17.3) as $F_x = \rho a V^2 \sin^2 \theta$

As the force is given in Newton, the value of ρ should be taken equal to 1000 kg/m^3

∴ $1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30° = .05 V^2$

∴ $V^2 = \frac{150}{.05} = 3000.0$

$V = 54.77 \text{ m/s}$

∴ Discharge, $Q = \text{Area} \times \text{Velocity}$

$= .001963 \times 54.77 = 0.1075 \text{ m}^3\text{/s} = \mathbf{107.5 \text{ liters/s. Ans.}}$

Problem 17.5 *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

Solution. Given :

Diameter of the jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

Velocity of jet, $V = 40 \text{ m/s}$

Angle of deflection $= 120°$

From equation [17.6 (*A*)], the angle of deflection $= 180° - \theta$

∴ $180° - \theta = 120°$ or $\theta = 180° - 120° = 60°$

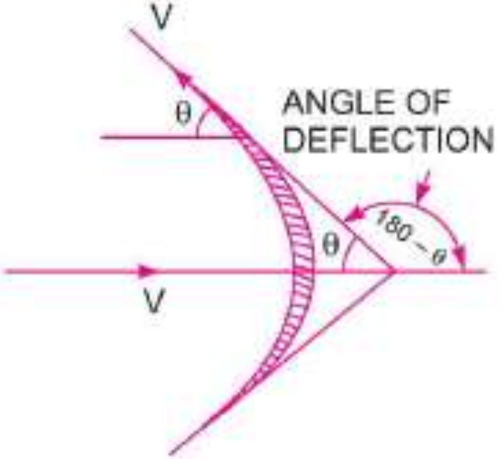


Fig. 17.5

Force exerted by the jet on the curved plate in the direction of the jet is given by equation (17.5) as

$$F_x = \rho a V^2 [1 + \cos\theta]$$

$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \textbf{4711.15 N. Ans.}$$

FORCE OF JETS ON MOVING PLATE:

The following cases of the moving plates will be considered :

1. Flat vertical plate moving in the direction of the jet and away from the jet,
2. Inclined plate moving in the direction of the jet, and
3. Curved plate moving in the direction of the jet or in the horizontal direction.

17.4.1 Force on Flat Vertical Plate Moving in the Direction of Jet. Fig. 17.10 shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the jet (absolute),

a = Area of cross-section of the jet,

u = Velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence relative velocity of the jet with respect to plate

$$= (V - u)$$

Mass of water striking the plate per sec

= ρ × Area of jet × Velocity with which jet strikes the plate

$$= \rho a \times [V - u]$$

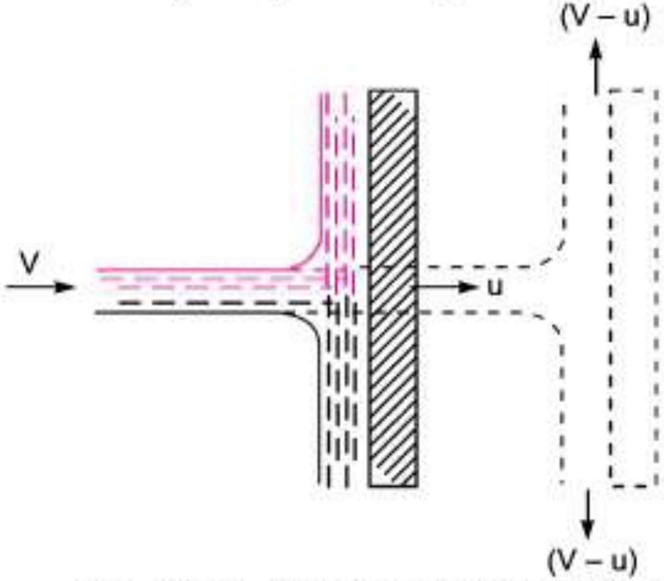


Fig. 17.10 *Jet striking a flat vertical moving plate.*

∴ Force exerted by the jet on the moving plate in the direction of the jet,

F_x = Mass of water striking per sec × [Initial velocity with which water strikes – Final velocity]

$$= \rho a(V - u)\,[(V - u) - 0] \qquad (\because \text{ Final velocity in the direction of jet is zero})$$

$$= \rho a(V - u)^2 \qquad \text{...(17.11)}$$

In this case, the work will be done by the jet on the plate, as plate is moving. For the stationary plates, the work done is zero.

∴ Work done per second by the jet on the plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force.}}{\text{Time}}$$

$$= F_x \times u = \rho a(V - u)^2 \times u \qquad \text{...(17.12)}$$

In equation (17.12), if the value of ρ for water is taken in S.I. units (*i.e.*, 1000 kg/m^3), the work done will be in N m/s. The term $\frac{\text{'Nm'}}{\text{s}}$ is equal to W (watt).

17.4.2 Force on the Inclined Plate Moving in the Direction of the Jet. Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.11.

Let V = Absolute velocity of jet of water,

u = Velocity of the plate in the direction of jet,

a = Cross-sectional area of jet, and

θ = Angle between jet and plate.

Relative velocity of jet of water = $(V - u)$

∴ The velocity with which jet strikes = $(V - u)$

Mass of water striking per second

$$= \rho \times a \times (V - u)$$

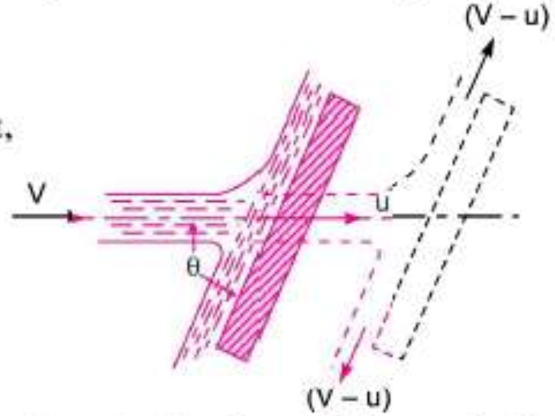


Fig. 17.11 *Jet striking an inclined moving plate.*

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V - u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

F_n = Mass striking per second × [Initial velocity in the normal direction with which jet strikes – Final velocity]

$$= \rho a (V - u) [(V - u) \sin\theta - 0] = \rho a (V - u)^2 \sin\theta \quad ...(17.13)$$

This normal force F_n is resolved into two components namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$\therefore \quad F_x = F_n \sin\theta = \rho a (V - u)^2 \sin^2\theta \quad ...(17.14)$$

$$F_y = F_n \cos\theta = \rho a (V - u)^2 \sin\theta \cos\theta \quad ...(17.15)$$

∴ Work done per second by the jet on the plate

= F_x × Distance per second in the direction of x

$$= F_x \times u = \rho a (V - u)^2 \sin^2\theta \times u = \rho a (V - u)^2 u \sin^2\theta \text{ N m/s.} \quad ...(17.16)$$

Problem 17.11 *A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find:*

(i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate per second.

Solution. Given :

Diameter of the jet, $d = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15$ m/s

Velocity of the plate, $u = 6$ m/s.

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation (17.11),

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times .007854 \times (15 - 6)^2 \text{ N} = \mathbf{636.17 \text{ N. Ans.}}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02 \text{ Nm/s. Ans.}}$$

Problem 17.12 *For Problem 17.11, find the power and efficiency of the jet.*

Solution. The given data from Problem 17.11 is

$$a = .007854 \text{ m}^2, V = 15 \text{ m/s}, u = 6 \text{ m/s}$$

Also work done per second by the jet = 3817.02 Nm/s

(*i*) Power of the jet in kW $= \frac{\text{Work done per second}}{1000} = \frac{3817.02}{1000} = \mathbf{3.81}$

(*ii*) Efficiency of the jet (η) $= \frac{\text{Output of the jet per second}}{\text{Input of the jet per second}}$

where output of jet/sec = Work done by jet per second = 3817.02 Nm/s

input per second = Kinetic energy of the jet/sec

$$= \frac{1}{2}\left(\frac{\text{mass}}{\text{sec}}\right) V^2 = \frac{1}{2}(\rho a V) \times V^2 = \frac{1}{2}\rho a V^3$$

$$= \frac{1}{2} \times 1000 \times .007854 \times 15^3 \text{ Nm/s} = 13253.6 \text{ Nm/s}$$

$$\eta \text{ of the jet} = \frac{3817.02}{13253.6} = 0.288 = \mathbf{28.8\%. \text{ Ans.}}$$

Problem 17.13 *A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary , and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.*

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Angle between the jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet, $V = 30$ m/s.

(*i*) When the plate is stationary, the normal force on the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta = 1000 \times .004417 \times 30^2 \times \sin 45^\circ = \mathbf{2810.96\ N.\ Ans.}$$

(*ii*) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by equation (17.13) as

$$F_n = \rho a (V - u)^2 \sin\theta \qquad \text{where } u = 15 \text{ m/s.}$$

$$= 1000 \times .004417 \times (30 - 15)^2 \times \sin 45^\circ = \mathbf{702.74\ N.\ Ans.}$$

(*iii*) The power and efficiency of the jet when plate is moving is obtained as

Work done per second by the jet

= Force in the direction of jet × Distance moved by the plate in the direction of jet/sec

$= F_x \times u$, where $F_x = F_n \sin\theta = 702.74 \times \sin 45^\circ = 496.9$ N

Work done/sec $= 496.9 \times 15 = 7453.5$ Nm/s

$$\therefore \quad \text{Power in kW} = \frac{\text{Work done per second}}{1000} = \frac{7453.5}{1000} = \mathbf{7.453\ kW.\ Ans.}$$

$$\text{Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$$

$$= \frac{7453.5}{\frac{1}{2}(\rho a V) \times V^2} = \frac{7453.5}{\frac{1}{2}\rho a V^3} = \frac{7453.5}{\frac{1}{2} \times 1000 \times .004417 \times 30^3}$$

$$= 0.1249 \simeq 0.125 = \mathbf{12.5\%.\ Ans.}$$

17.4.3 Force on the Curved Plate when the Plate is Moving in the Direction of Jet. Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.12.

Let V = Absolute velocity of jet,

a = Area of jet,

u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate = $(V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane = $(V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of the velocity in the direction of jet

$$= -(V - u)\cos\theta$$

(–ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet = $(V - u)\sin\theta$.

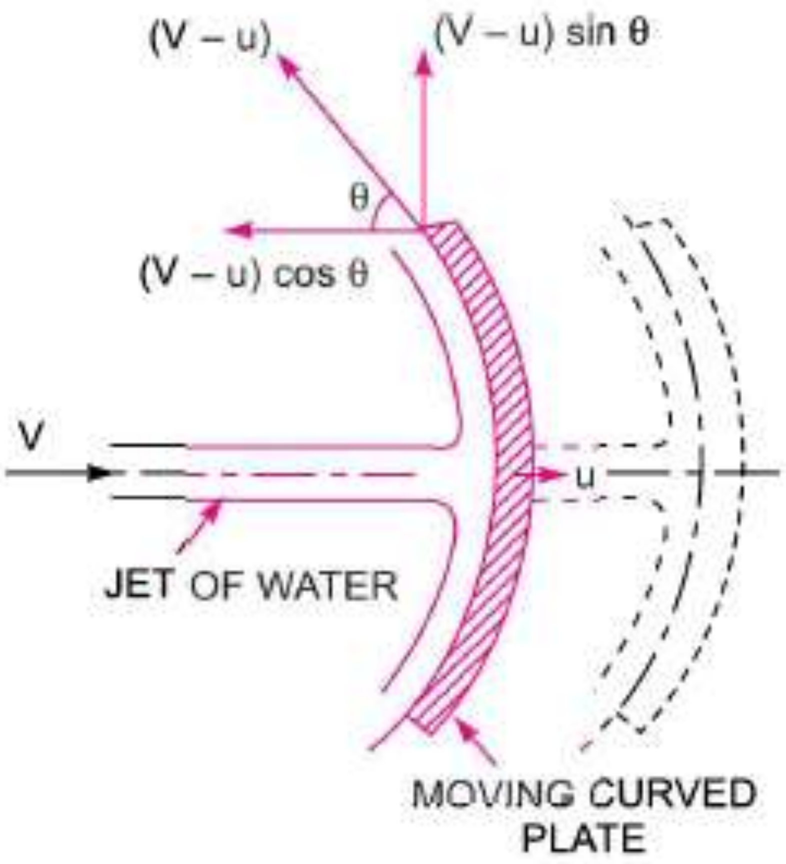


Fig. 17.12 *Jet striking a curved moving plate.*

Mass of the water striking the plate = $\rho \times a \times$ Velocity with which jet strikes the plate

$$= \rho a(V - u)$$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$= \rho a(V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$

$$= \rho a(V - u)\,[(V - u) + (V - u)\cos\theta\,]$$

$$= \rho a(V - u)^2\,[1 + \cos\theta\,] \qquad \ldots(17.17)$$

Work done by the jet on the plate per second

$= F_x \times$ Distance travelled per second in the direction of x

$$= F_x \times u = \rho a(V - u)^2\,[1 + \cos\theta\,] \times u$$

$$= \rho a(V - u)^2 \times u\,[1 + \cos\theta\,] \qquad \ldots(17.18)$$

Problem 17.14 *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \dfrac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20$ m/s

Velocity of the plate, $u = 8$ m/s

Angle of deflection of the jet, $= 165°$

$\therefore$ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation (17.17) as

$$= F_x = \rho a(V - u)^2\,(1 + \cos\theta)$$

$= 1000 \times .004417 \times (20 - 8)^2\,[1 + \cos 15°] =$ **1250.38 N. Ans.**

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04 \text{ N m/s}$$

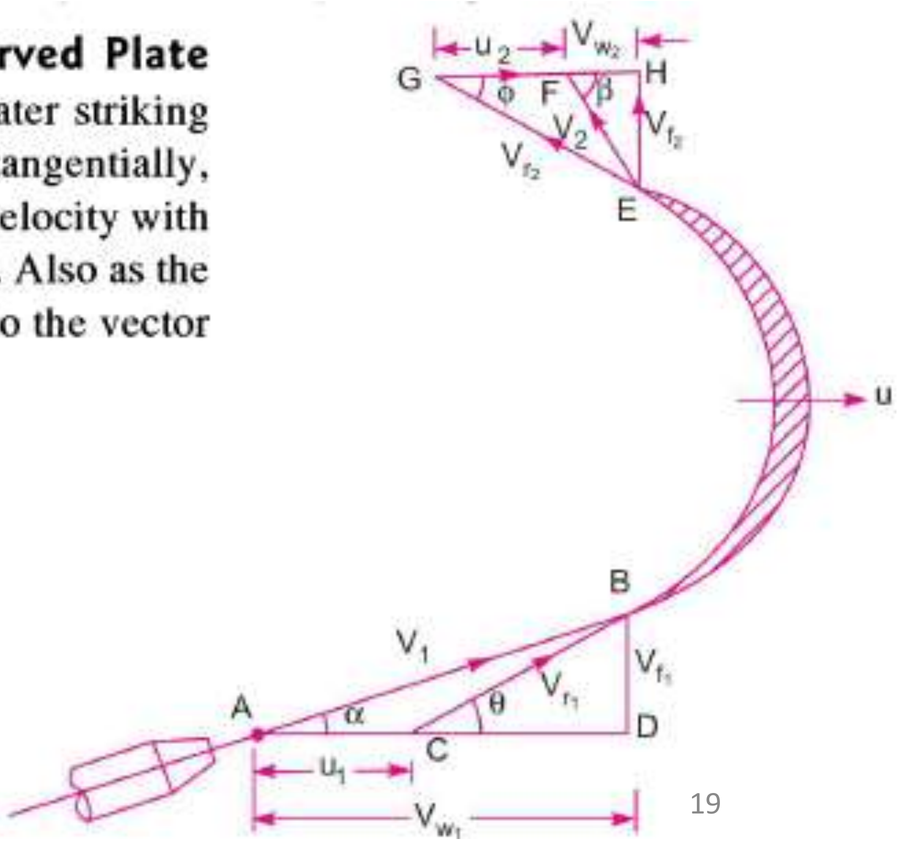
$\therefore$ Power of the jet $= \dfrac{10003.04}{1000} =$ **10 kW. Ans.**

(*iii*) Efficiency of the jet $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.564 = \mathbf{56.4\%.\ Ans}$$

17.4.4 Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the Tips. Fig. 17.15 shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.



V_1 = Velocity of the jet at inlet.

u_1 = Velocity of the plate (vane) at inlet.

V_{r_1} = Relative velocity of jet and plate at inlet.

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.

θ = Angle made by the relative velocity (V_{r_2}) with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.

V_{w_1} = It is also known as velocity of whirl at inlet.

V_{f_1} = It is also known as velocity of flow at inlet.

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = Velocity of the vane at outlet.

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet.

V_{w_1} and V_{f_1} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} = It is also called the velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

The triangles ABD and EGH are called the velocity triangles at inlet and outlet. These velocity triangles are drawn as given below :

1. Velocity Triangle at Inlet. Take any point A and draw a line $AB = V_1$ in magnitude and direction which means line AB makes an angle α with the horizontal line AD. Next draw a line $AC = u_1$ in magnitude. Join C to B. Then CB represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D.

Then BD = Represents the velocity of flow at inlet = V_{f_1}

AD = Represents the velocity of whirl at inlet = V_{w_1}

$\angle BCD$ = Vane angle at inlet = θ.

2. Velocity Triangle at Outlet. If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity equal to V_{r_1} and will come out of the vane with a relative relocity V_{r_2}. This means that the relative velocity at outlet $V_{r_2} = V_{r_1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw EG in the tangential direction of the vane at outlet and cut $EG = V_{r_2}$. From G, draw a line GF in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet. Join EF. Then EF represents the absolute velocity of the jet at outlet in magnitude and direction. From E draw a vertical line EH to meet the line GF produced at H. Then

EH = Velocity of flow at outlet = V_{f_2}

FH = Velocity of whirl at outlet = V_{w_2}

ϕ = Angle of vane at outlet.

β = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have

$u_1 = u_2 = u$ = Velocity of vane in the direction of motion and

$V_{r_1} = V_{r_2}$.

Now mass of water striking vane per sec = $\rho a V_{r_1}$ ...(*i*)

where a = Area of jet of water, V_{r_1} = Relative velocity at inlet.

$\therefore$ Force exerted by the jet in the direction of motion

F_x = Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion – Final velocity of jet in the direction of motion] ...(*ii*)

But initial velocity with which jet strikes the vane = V_{r_1}

The component of this velocity in the direction of motion

$$= V_{r_1} \cos\theta = (V_{w_1} - u_1) \qquad \text{(See Fig. 17.15)}$$

Similarly, the component of the relative velocity at outlet in the direction of motion = $-V_{r_2}\cos\phi$

$$= -[u_2 + V_{w_2}]$$

–ve sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Substituting the equation (*i*) and all above values of the velocities in equation (*ii*), we get

$$F_x = \rho a V_{r_1}[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}] = \rho a V_{r_1}[V_{w_1} - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_{r_1}[V_{w_1} + V_{w_2}] \qquad (\because \ u_1 = u_2) \quad \text{...(iii)}$$

Equation (*iii*) is true only when angle β shown in Fig. 17.15 is an acute angle. If $\beta = 90°$, the $V_{w_2} = 0$, then equation (*iii*) becomes as,

$$F_x = \rho a V_{r_1}[V_{w_1}]$$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}]$$

Thus in general, F_x is written as $F_x = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}]$ ...(17.19)

Work done per second on the vane by the jet

= Force × Distance per second in the direction of force

$$= F_x \times u = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u \qquad \text{...(17.20)}$$

∴ Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Weight of fluid striking/s}} \frac{\text{Nm/s}}{\text{N/s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{g \times \rho a V_{r_1}} = \text{Nm/N}$$

$$= \frac{1}{g}[V_{w_1} \pm V_{w_2}] \times u \text{ Nm/N} \qquad \text{...(17.21)}$$

Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Mass of fluid striking / s}} \frac{\text{Nm / s}}{\text{kg / s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\rho a V_{r_1}} \text{ Nm/kg}$$

$$= (V_{w_1} \pm V_{w_2}) \times u \text{ Nm/kg} \qquad \text{...[17.21(A)]}$$

Note. Equation (17.21) gives the work done per unit weight whereas equation [17.21(*A*)] gives the work done per unit mass.

3. Efficiency of Jet. The work done by the jet on the vane given by equation (17.20), is the output of the jet whereas the initial kinetic energy of the jet is the input. Hence, the efficiency (η) of the jet is expressed as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second on the vane}}{\text{Initial K. E. per second of the jet}} = \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} m V_1^2}$$

where m = mass of the fluid per second in the jet = $\rho a V_1$

V_1 = initial velocity of jet

$$\therefore \qquad \eta = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2}. \qquad \text{...[17.21(B)]}$$

Problem 17.18 *A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane an outlet. Calculate :*

(i) Vane angles, so that the water enters and leaves the vane without shock.

(ii) Work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.

Solution. Given :

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

Angle made by jet at inlet, with direction of motion of vane,

$$\alpha = 20°$$

Angle made by the leaving jet, with the direction of motion

$$= 130°$$

$\therefore$ $\beta = 180° - 130° = 50°$

In this problem, $u_1 = u_2 = 10$ m/s

$$V_{r_1} = V_{r_2}$$

(*i*) **Vane Angle** means angle made by the relative velocities at inlet and outlet, *i.e.*, θ and ϕ.

From Fig. 17.16, in ΔABD, we have $\tan\theta = \dfrac{BD}{CD}$

$$= \frac{V_{f_1}}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1} \quad ...(i)$$

where $V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20° = 6.84$ m/s

$V_{w_1} = V_1 \cos\alpha = 20 \times \cos 20° = 18.794$ m/s.

$u_1 = 10$ m/s

$$\therefore \quad \tan\theta = \frac{6.84}{18.794 - 10} = .7778 \text{ or } \theta = 37.875°$$

$$\therefore \quad \theta = \mathbf{37° \; 52.5'. \; Ans.}$$

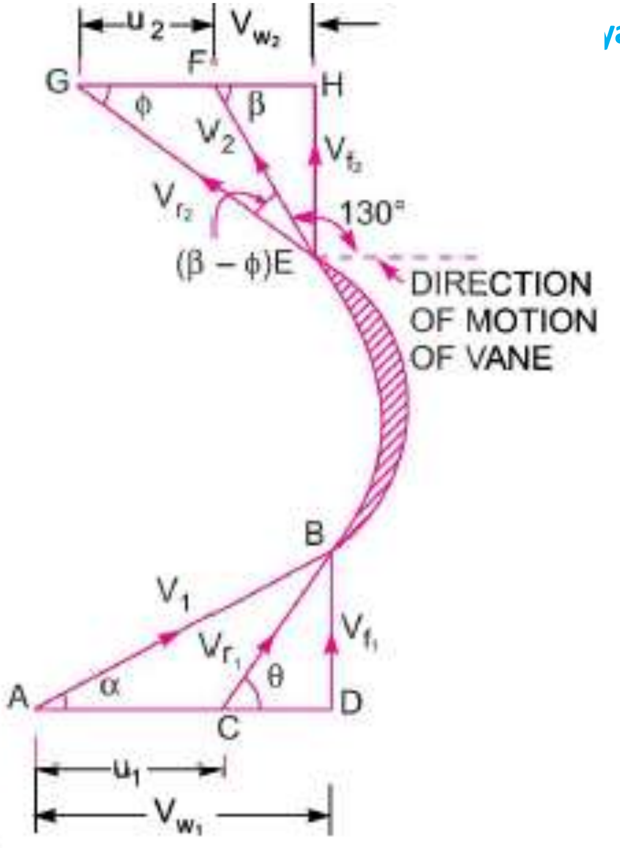


Fig. 17.16

From, ΔABC, $\sin\theta = \frac{V_{f_1}}{V_{r_1}}$ or $V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875^\circ} = 11.14$

$\therefore$ $V_{r_2} = V_{r_1} = 11.14$ m/s.

From, ΔEFG, applying sine rule, we have

$$\frac{V_{r_2}}{\sin(180^\circ - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

or $$\frac{11.14}{\sin\beta} = \frac{10}{\sin[\beta - \phi]} \quad \text{or} \quad \frac{11.14}{\sin 50^\circ} = \frac{10}{\sin[50^\circ - \phi]} \qquad (\because \ \beta = 50^\circ)$$

$\therefore$ $$\sin(50^\circ - \phi) = \frac{10 \times \sin 50^\circ}{11.14} = 0.6876 = \sin 43.44^\circ$$

$\therefore$ $50^\circ - \phi = 43.44^\circ$ or $\phi = 50^\circ - 43.44^\circ = 6.56^\circ$

$\therefore$ $\phi = 6^\circ\ 33.6'$. **Ans.**

(*ii*) Work done per second per unit weight of the water striking the vane per second is given by equation (17.21) as

$$= \frac{1}{g}[V_{w_1} + V_{w_2}] \times u \text{ Nm/N (+ ve sign is taken as } \beta \text{ is an acute angle)}$$

where $V_{w_1} = 18.794$ m/s, $V_{w_2} = GH - GF = V_{r_2}\cos\phi - u_2 = 11.14 \times \cos 6.56^\circ - 10 = 1.067$ m/s

$u = u_1 = u_2 = 10$ m/s

$\therefore$ Work done per unit weight of water

$$= \frac{1}{9.81}[18.794 + 1.067] \times 10 \text{ Nm/N} = \mathbf{20.24 \text{ Nm/N. Ans.}}$$

Problem 17.19 *A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.*

Velocity of jet, $V_1 = 40$ m/s
Velocity of vane, $u_1 = 20$ m/s
Angle made by jet at inlet, $\alpha = 30°$
Angle made by leaving jet $= 90°$
$\therefore$ $\beta = 180° - 90° = 90°$
For this problem, we have

$u_1 = u_2 = u = 20$ m/s

Vane angles at inlet and outlet are θ and ϕ respectively.
From ΔBCD, we have

$$\tan\theta = \frac{BD}{CD} = \frac{BD}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1}$$

where $V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30° = 20$ m/s

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30° = 34.64$ m/s

$u_1 = 20$ m/s

$$\therefore \quad \tan\theta = \frac{20}{34.64 - 20} = \frac{20}{14.64} = 1.366 = \tan 53.79°$$

$\therefore$ $\theta = 53.79°$ or **53° 47.4′. Ans.**

Also from ΔBCD,

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \quad \text{or} \quad V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{20}{\sin 53.79°}$$

$\therefore$ $V_{r_1} = 24.78$

But $V_{r_2} = V_{r_1} = 24.78$

Hence, from ΔEFG, $\cos\phi = \dfrac{u_2}{V_{r_2}} = \dfrac{20}{24.78} = 0.8071 = \cos 36.18°$

$\therefore$ $\phi = 36.18°$ or **36° 10.8′. Ans.**

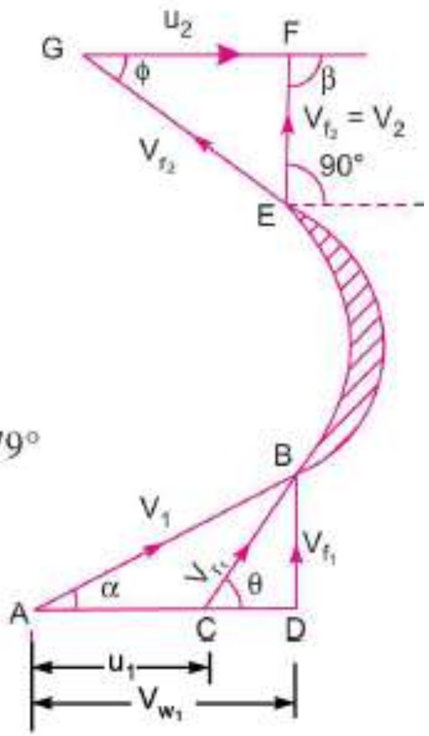


Fig. 17.17

Problem 17.20 *A jet of water of diameter 50 mm, having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet. Determine :*

(i) The force exerted by the jet on the vane in the direction of motion.

(ii) Work done per second by the jet.

Solution. Given :

Diameter of the jet, $d = 50\text{ mm} = 0.05\text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.05)^2 = .001963\text{ m}^2$

Velocity of jet, $V_1 = 20\text{ m/s}$

Velocity of vane, $u_1 = 10\text{ m/s}$

As jet and vane are moving in the same direction,

$\therefore \quad \alpha = 0$

Angle made by the leaving jet, with the direction of motion = 60°

$\therefore \quad \beta = 180° - 60° = 120°$

For this problem, we have

$$u_1 = u_2 = u = 10\text{ m/s}$$

$$V_{r_1} = V_{r_2}$$

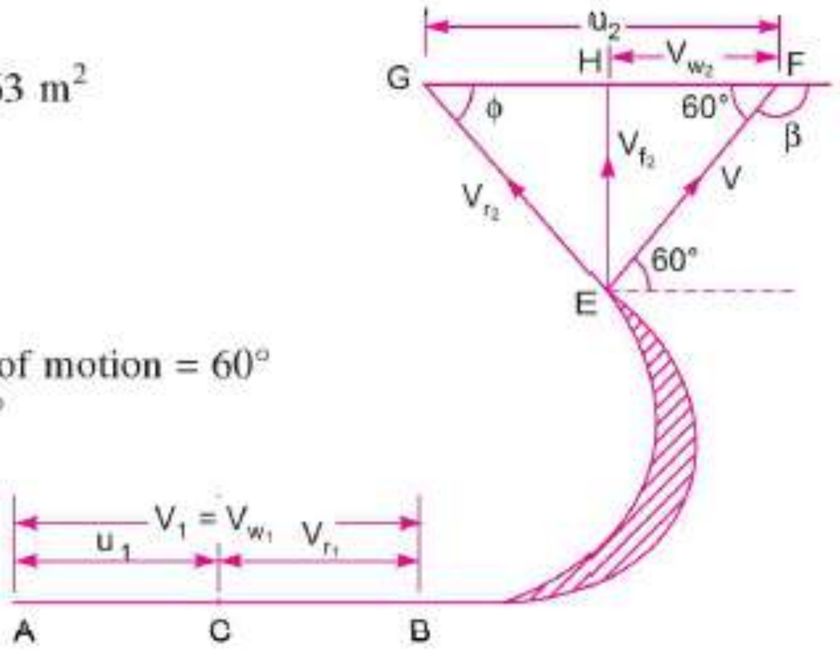


Fig. 17.18

From Fig. 17.18, we have

$$V_{r_1} = AB - AC = V_1 - u_1$$

$$= 20 - 10 = 10\text{ m/s}$$

$$V_{w_1} = V_1 = 20\text{ m/s}$$

$$\therefore \quad V_{r_2} = V_{r_1} = 10\text{ m/s}$$

Now in ΔEFG, $EG = V_{r_2} = 10\text{ m/s}$,

$GF = u_2 = 10\text{ m/s}$

$\angle GEF = 180° - (60° + \phi) = (120° - \phi)$

From sine rule, we have

$$\frac{EG}{\sin 60°} = \frac{GF}{\sin(120° - \phi)} \quad \text{or} \quad \frac{10}{\sin 60°} = \frac{10}{\sin(120° - \phi)}$$

or $\sin 60° = \sin(120° - \phi)$

$\therefore \quad 60° = 120° - \phi \quad \text{or} \quad \phi = 120° - 60° = 60°$

Now $V_{w_2} = HF = GF - GH$

$$= u_2 - V_{r_2}\cos\phi = 10 - 10 \times \cos 60° = 10 - 5 = 5\text{ m/s}$$

(*i*) The force exerted by the jet on the vane in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_{r_1}\,[V_{w_1} - V_{w_2}] \quad \text{(–ve sign is taken as } \beta \text{ is an obtuse angle)}$$

$$= 1000 \times .001963 \times 10\,[20 - 5]\ \text{N} = \mathbf{294.45\ N.\ \ Ans.}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 294.45 \times 10 = 2944.5\ \text{N m/s}$$

$$= \mathbf{2944.5\ W.\ \ Ans.} \qquad [\because\ \text{Nm/s} = \text{W (watt)}]$$

17.4.5 Force Exerted by a Jet of Water on a Series of Vanes. The force exerted by a jet of water on a *single* moving plate (which may be flat or curved) is not practically feasible. This case is only a theoretical one. In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in Fig. 17.22. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

V = Velocity of jet,
d = Diameter of jet,
a = Cross-sectional area of jet,
$= \dfrac{\pi}{4} d^2$
u = Velocity of vane.

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of plates = $\rho a V$.

Also the jet strikes the plate with a velocity = $(V - u)$.

After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

∴ The force exerted by the jet in the direction of motion of plate,

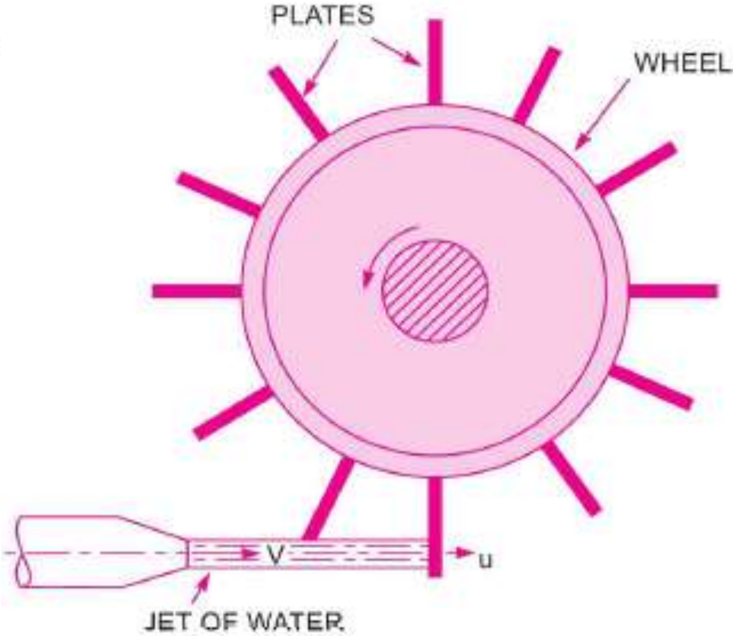
$$F_x = \text{Mass per second [Initial velocity – Final velocity]}$$

$$= \rho a V[(V - u) - 0] = \rho a V[V - u] \qquad \text{...(17.22)}$$

Work done by the jet on the series of plates per second

$$= \text{Force} \times \text{Distance per second in the direction of force}$$

$$= F_x \times u = \rho a V[V - u] \times u$$



Kinetic energy of the jet per second

$$= \frac{1}{2} mV^2 = \frac{1}{2} (\rho a V) \times V^2 = \frac{1}{2} \rho a V^3$$

$\therefore$ Efficiency,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho a V[V-u] \times u}{\frac{1}{2}\rho a V^3} = \frac{2u[V-u]}{V^2} \quad ...(17.23)$$

Condition for Maximum Efficiency. Equation (17.23) gives the value of the efficiency of the wheel. For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0$$

or

$$\frac{2V - 2 \times 2u}{V^2} = 0 \quad \text{or} \quad 2V - 4u = 0 \quad \text{or} \quad V = \frac{4u}{2} = 2u \quad \text{or} \quad u = \frac{V}{2}. \quad ...(17.24)$$

Maximum Efficiency. Substituting the value of $V = 2u$ in equation (17.23), we get the maximum efficiency as

$$\eta_{max} = \frac{2u[2u-u]}{(2u)^2} = \frac{2u \times u}{2u \times 2u} = \frac{1}{2} = 0.5 \text{ or } 50\%. \quad ...(17.25)$$

17.4.6 Force Exerted on a Series of Radial Curved Vanes. For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal. Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 17.23. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

R_1 = Radius of wheel at inlet of the vane,
R_2 = Radius of the wheel at the outlet of the vane,
ω = Angular speed of the wheel.
$u_1 = \omega R_1$ and $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig. 17.23.

The mass of water striking per second for a series of vanes

= Mass of water coming out from nozzle per second

= $\rho a V_1$, where a = Area of jet and V_1 = Velocity of jet.

Momentum of water striking the vanes in the tangential direction per sec at inlet

= Mass of water per second × Component of V_1 in the tangential direction

= $\rho a V_1 \times V_{w_1}$ ($\because$ Component of V_1 in tangential direction = $V_1 \cos \alpha = V_{w_1}$

Similarly, momentum of water at outlet per sec

= $\rho a V_1$ × Component of V_2 in the tangential direction

= $\rho a V_1 \times (-V_2 \cos \beta) = -\rho a V_1 \times V_{w_2}$ ($\because$ $V_2 \cos \beta = V_{w_2}$

–ve sign is taken as the velocity V_2 at outlet is in opposite direction.

Now, angular momentum per second at inlet

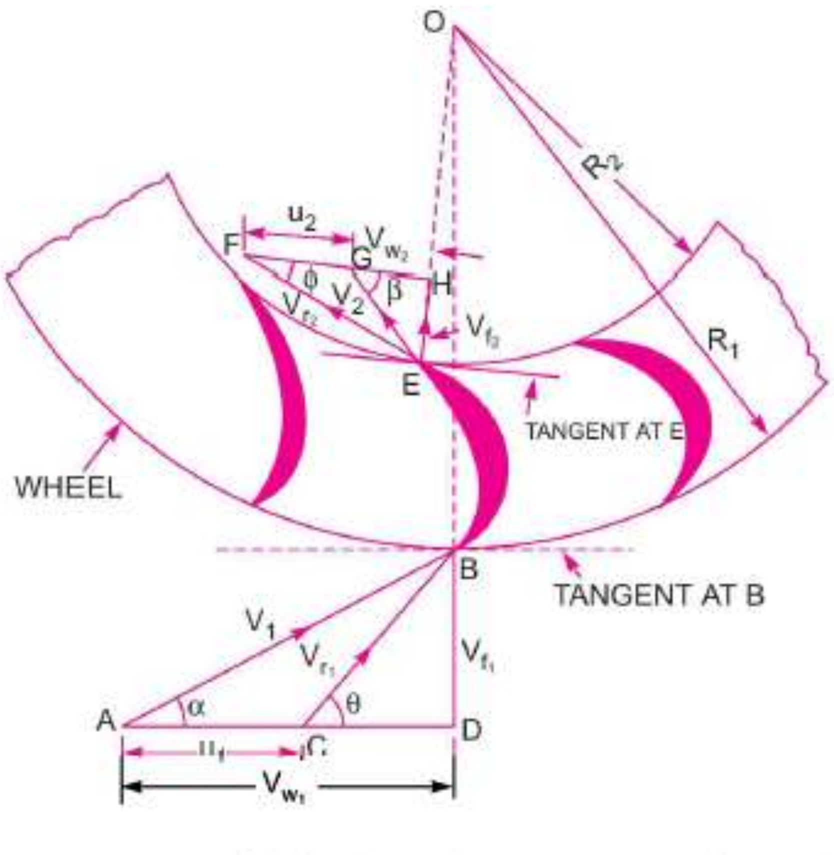
= Momentum at inlet × Radius at inlet

= $\rho a V_1 \times V_{w_1} \times R_1$

Angular momentum per second at outlet

= Momentum of outlet × Radius at outlet

= $-\rho a V_1 \times V_{w_2} \times R_2$



g. 17.23 *Series of radial curved vanes mounted on a wheel.*

Torque exerted by the water on the wheel,

T = Rate of change of angular momentum

= [Initial angular momentum per second – Final angular momentum per second]

$= \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2]$

Work done per second on the wheel

$$= \text{Torque} \times \text{Angular velocity} = T \times \omega$$

$$= \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2] \times \omega = \rho a V_1 [V_{w_1} \times R_1 \times \omega + V_{w_2} R_2 \times \omega]$$

$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} \times u_2] \quad (\because \; u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

If the angle β in Fig. 17.23 is an obtuse angle then work done per second will be given as

$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$$

$\therefore$ The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \text{...(17.26)}$$

If the discharge is radial at outlet, then $\beta = 90°$ and work done becomes as

$$= \rho a V_1 [V_{w_1} u_1] \quad (\because \; V_{w_2} = 0) \quad \text{...(17.27)}$$

Efficiency of the Radial Curved Vane

The work done per second on the wheel is the output of the system whereas the initial kinetic energy per second of the jet is the input. Hence, efficiency of the system is expressed as

$$\text{Efficiency,} \quad \eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{1}{2}(\text{mass/sec}) \times V_1^2}$$

$$= \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{V_1^2}. \quad \text{...(17.28)}$$

If there is no loss of energy when water is flowing over the vanes, the work done on the wheel per second is also equal to the change in kinetic energy of the jet per second. Hence, the work done per second on the wheel is also given as

Work done per second on the wheel

$$= \text{Change of K.E. per second of the jet}$$

$$= (\text{Initial K.E. per second} - \text{Final K.E. per second}) \text{ of the jet}$$

$$= \left(\frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2\right)$$

$$= \frac{1}{2} m \left(V_1^2 - V_2^2\right) = \frac{1}{2}\left(\rho a V_1^2\right)\left(V_1^2 - V_2^2\right) \qquad (\because \text{mass/second} = \rho a V_1)$$

$$\text{Hence efficiency, } \eta = \frac{\text{Work done per second on the wheel}}{\text{Initial K.E. per second of the jet}}$$

$$= \frac{\frac{1}{2}\rho a V_1^2 \left(V_1^2 - V_2^2\right)}{\frac{1}{2}\left(\rho a V_1^2\right) \cdot V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2} = \left(1 - \frac{V_2^2}{V_1^2}\right) \qquad \text{...(17.28A)}$$

From the above equation, it is clear that for a given initial velocity of the jet (*i.e.*, V_1), the efficiency will be maximum, when V_2 is minimum. But V_2 cannot be zero as in that case the incoming jet will not move out of the vane. Equation (17.28) also gives the efficiency of the system. From this equation, it is clear that efficiency will be maximum when V_{w_2} is added to V_{w_1}. This is only posible if β is an acute* angle. Also for maximum efficiency V_{w_2} should also be maximum. This is only possible if $\beta = 0$. In that case $V_{w_2} = V_2$ and angle ϕ will be zero. But in actual practice ϕ cannot be zero. Hence for maximum efficiency, the angle ϕ should be minimum.

Problem 17.25 *A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find :*

(a) the angles of vanes tips so that water enters and leaves without shock,
(b) the work done per unit weight of water entering the vanes, and
(c) the efficiency.

Velocity of jet, $V_1 = 35$ m/s

Velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes = 120°

∴ Angle $\beta = 180° - 120° = 60°$

(*a*) *Angle of vanes tips.*

From inlet velocity triangle

$$V_{w_1} = V_1 \cos\alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

∴ $\theta = \tan^{-1} 1.697 = \mathbf{60°. \ Ans.}$

By sine rule, $\dfrac{V_{r_1}}{\sin 90°} = \dfrac{V_{f_1}}{\sin\theta}$ or $\dfrac{V_{r_1}}{1} = \dfrac{17.50}{\sin 60°}$

∴ $V_{r_1} = \dfrac{17.50}{.866} = 20.25$ m/s.

Now, $V_{r_2} = V_{r_1} = 20.25$ m/s

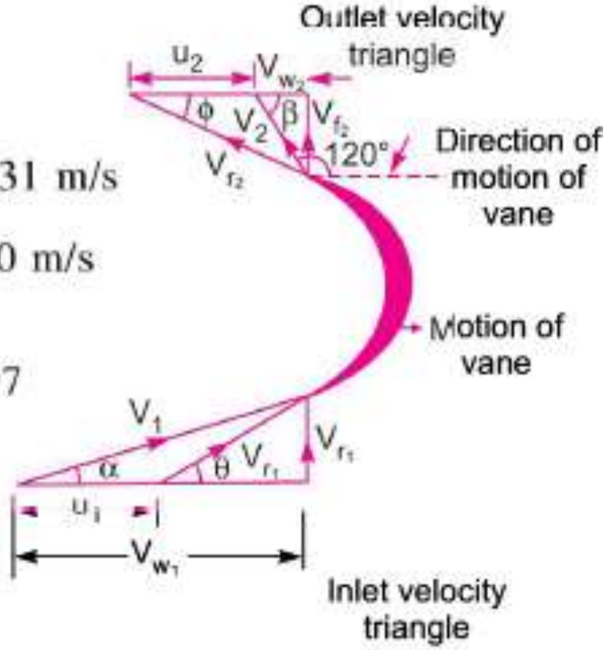


Fig. 17.23(*a*)

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 120°} = \frac{u_2}{\sin(60° - \phi)} \quad \text{or} \quad \frac{20.25}{0.886} = \frac{20}{\sin(60° - \phi)}$$

$$\therefore \quad \sin(60° - \phi) = \frac{20 \times 0.866}{20.25} = 0.855 = \sin(58.75°)$$

$$60° - \phi = 58.75°$$

$$\therefore \quad \phi = 60° - 58.75° = \mathbf{1.25°.\ Ans.}$$

(*b*) *Work done per unit weight of water entering* $= \frac{1}{g}(V_{w_1} + V_{w_2}) \times u_1$

$$V_{w_1} = 30.31 \text{ m/s and } u_1 = 30 \text{ m/s}$$

The value of V_{w_2} is obtained from outlet velocity triangle

$$V_{w_2} = V_{r_2} \cos\phi - u_2 = 20.25 \cos 1.25° - 20.0 = 0.24 \text{ m/s}$$

$$\therefore \quad \text{Work done/unit weight} = \frac{1}{9.81}[30.31 + 0.24] \times 20 = \mathbf{62.28\ Nm/N.\ Ans.}$$

(*c*) *Efficiency*

$$= \frac{\text{Work done per kg}}{\text{Energy supplied per kg}}$$

$$= \frac{62.28}{\frac{V_1^2}{2g}} = \frac{62.28 \times 2 \times 9.81}{35 \times 35} = \mathbf{99.74\%\ Ans.}$$

Problem 17.26 *A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 r.p.m. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine :*

(i) Vane angles at inlet and outlet, *(ii) Work done per unit weight of water, and*
(iii) Efficiency of the wheel.

Velocity of jet, $V_1 = 30$ m/s

Speed of wheel, $N = 200$ r.p.m.

∴ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94$ rad/s

Angle of jet at inlet, $\alpha = 20°$

Velocity of jet at outlet, $V_2 = 5$ m/s

Angle made by the jet at outlet with the tangent to wheel = 130°

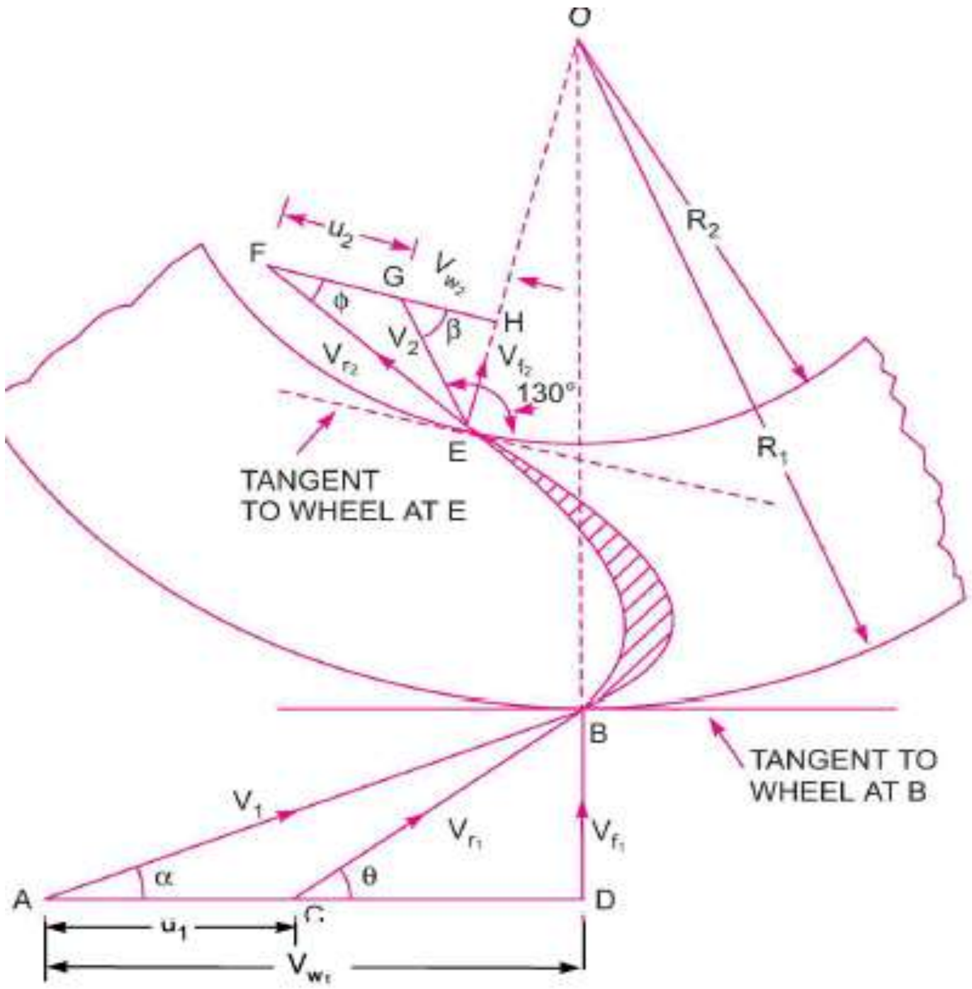
∴ Angle, $\beta = 180° - 130° = 50°$

Outer radius, $R_1 = 0.5$ m

Inner radius, $R_2 = 0.25$ m

∴ Velocity $u_1 = \omega \times R_1 = 20.94 \times 0.5 = 10.47$ m/s

And $u_2 = \omega \times R_2 = 20.94 \times 0.25 = 5.235$ m/s.



(*i*) **Vane angles** at inlet and outlet means the angle made by the relative velocities V_{r_1} and V_{r_2}, *i.e.*, angle θ and φ.

From ΔABD, $V_{w_1} = V_1 \cos\alpha = 30 \times \cos 20° = 28.19$ m/s

$V_{f_1} = V_1 \sin\alpha = 30 \times \sin 20° = 10.26$ m/s

In ΔCBD, $$\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{AD - AC} = \frac{10.26}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47} = 0.579 = \tan 30.07$$

∴ $\theta = 30.07°$ or **30° 4.2′. Ans.**

From outlet velocity Δ, $V_{w_2} = V_2 \cos\beta = 5 \times \cos 50° = 3.214$ m/s

$V_{f_2} = V_2 \times \sin\beta = 5 \sin 50° = 3.83$ m/s

In ΔEFH, $$\tan\phi = \frac{V_{f_2}}{u_2 + V_{w_2}} = \frac{3.83}{5.235 + 3.214} = 0.453 = \tan 24.385°$$

∴ $\phi = 24.385°$ or **24° 23.1′. Ans.**

(*ii*) Work done per second by water is given by equation (17.26)

$$= \rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2\right]$$

(+ ve sign is taken as β is acute angle in Fig.17.24)

∴ Work done* per second per unit weight of water striking per second

$$= \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\text{Weight of water/s}} = \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\rho a V_1 \times g}$$

$$= \frac{1}{g}\left[V_{w_1} u_1 + V_{w_2} u_2\right] \text{ Nm/N} = \frac{1}{9.81}\,[28.19 \times 10.47 + 3.214 \times 5.235$$

$$= \frac{1}{9.81}\,[295.15 + 16.82] = \mathbf{31.8\ Nm/N.\ Ans.}$$

η is given by equation (17.28) as

$$\eta = \frac{2\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{V_1^2} = \frac{2[28.19 \times 10.47 + 3.214 \times 5.235]}{30^2}$$

$$= \frac{2[295.15 + 16.82]}{30 \times 30} = 0.6932 \text{ or } \mathbf{69.32\%.\ Ans.}$$

INTRODUCTION TO HYDRAULIC MACHINERY

- Hydraulic Turbines
- Classification of Turbines – Impulse and Reaction Turbines,
- Pelton Wheel, Working Principle

CLASSIFICATION OF TURBINE

- ✓ **Hydraulic machine** are those machines which convert *hydraulic energy* (energy possessed by water) into *mechanical energy* (**turbine,** which is further converted into electrical energy) or *mechanical energy into hydraulic energy* (***pumps***).

  - ➢ Hydraulic machines are machinery and tools that use liquid fluid power to do simple work, operated by the use of hydraulics, where a liquid is the powering medium.

  - ➢ The hydraulic turbine is a prime mover that uses the energy of flowing water and converts is into the mechanical energy in the form of rotation of the runner.

Turbines consists of mainly study of Pelton turbine, Francis Turbine and Kaplan Turbine

GENERAL LAYOUT OF A HYDROELECTRIC POWER PLANT

(*i*) A dam constructed across a river to store water.

(*ii*) Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.

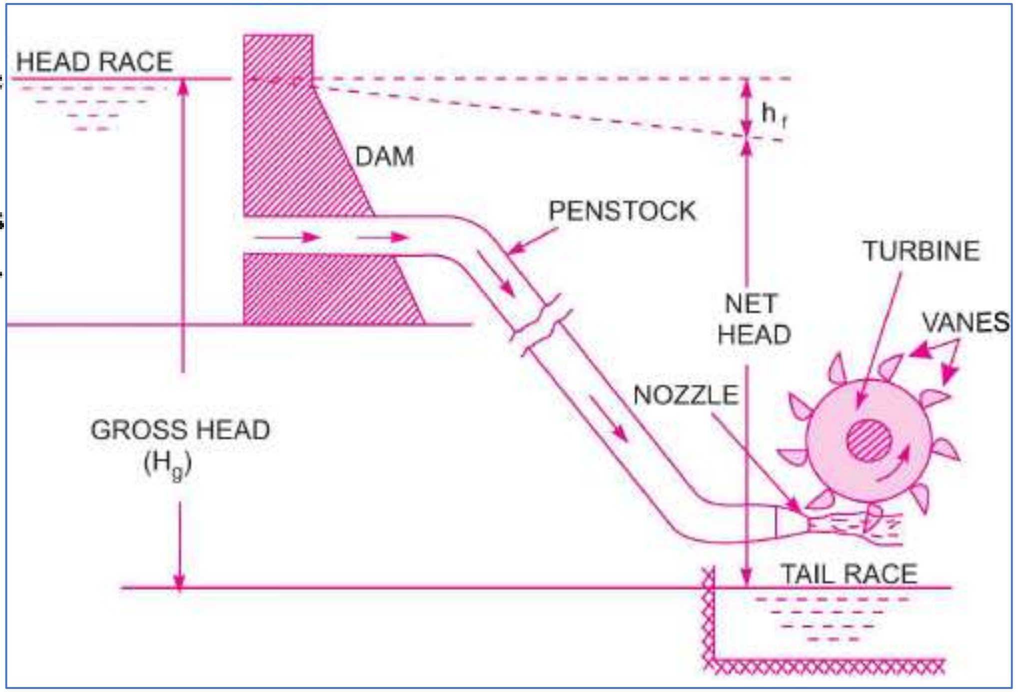
(*iii*) Turbines having different types of vanes fitted to the wheels.

(*iv*) Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race channel is also known as tail race.

1. Gross Head. The difference between the head race level and tail race level when no water is flowing is known as Gross Head. It is denoted by 'H_g' in Fig. 18.1.

2. Net Head. It is also called effective head and is defined as the head available at the inlet of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction between the water and penstocks occurs. Though there are other losses also such as loss due to bend, pipe fittings, loss at the entrance of penstock etc., yet they are having small magnitude as compared to head loss due to friction. If 'h_f' is the head loss due to friction between penstocks and water then net heat on turbine is given by

$$H = H_g - h_f \qquad \ldots(18.1)$$



General Layout of a Hydroelectric Power Plant

where H_g = Gross head, $h_f = \dfrac{4 \times f \times L \times V^2}{D \times 2g}$,

in which V = Velocity of flow in penstock,

L = Length of penstock,

D = Diameter of penstock.

3. Efficiencies of a Turbine. The following are the important efficiencies of a turbine.

(*a*) Hydraulic Efficiency, η_h (*b*) Mechanical Efficiency, η_m

(*c*) Volumetric Efficiency, η_v and (*d*) Overall Efficiency, η_o

(*a*) **Hydraulic Efficiency** (η_h). It is defined as the ratio of power given by water to the runner of a turbine (runner is a rotating part of a turbine and on the runner vanes are fixed) to the power supplied by the water at the inlet of the turbine. The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth. Hence, the power delivered to the runner of the turbine will be less than the power available at the inlet of the turbine. Thus, mathematically, the hydraulic efficiency of a turbine is written as

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}} \qquad \ldots(18.2)$$

where R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g} \frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \quad \text{...for Pelton Turbine}$$

$$= \frac{W}{g} \frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} \text{ kW} \quad \text{...for a radial flow turbine}$$

W.P. = Power supplied at inlet of turbine and also called water power

$$= \frac{W \times H}{1000} \text{ kW} \quad \text{...(18.3)}$$

where W = Weight of water striking the vanes of the turbine per second

$= \rho g \times Q$ in which Q = Volume of water/s,

V_{w_1} = Velocity of whirl at inlet,

V_{w_2} = Velocity of whirl at outlet,

u = Tangential velocity of vane,

u_1 = Tangential velocity of vane at inlet for radial vane,

u_2 = Tangential velocity of vane at outlet for radial vane,

H = Net head on the turbine.

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW} \quad \text{...(18.3}A\text{)}$$

For water $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad \text{W.P.} = \frac{1000 \times g \times Q \times H}{1000} = g \times Q \times H \text{ kW} \quad \text{...(18.3}B\text{)}$$

The relation (18.3*B*) is only used when the flowing fluid is water. If the flowing fluid is other than the water, then relation (18.3*A*) is used.

(*b*) **Mechanical Efficiency (η_m).** The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine. The ratio of the power available at the shaft of the turbine (known as S.P. or B.P.) to the power delivered to the runner is defined as mechanical efficiency. Hence, mathematically, it is written as

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}} \quad \text{...(18.4)}$$

(*c*) **Volumetric Efficiency (η_v).** The volume of the water striking the runner of a turbine is slightly less than the vloume of the water supplied to the turbine. Some of the volume of the water is discharged to the tail race without striking the runner of the turbine. Thus the ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency. It is written as

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}} \quad \text{...(18.5)}$$

(*d*) **Overall Efficiency (η_o).** It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine. It is written as :

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}} \qquad \text{(where R.P. = Power delivered to runner)}$$

$$= \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}}$$

$$= \eta_m \times \eta_h \qquad \left(\begin{array}{l} \because \text{From equation (18.4), } \dfrac{\text{S.P.}}{\text{R.P.}} = \eta_m \\ \text{and from equation (18.2), } \dfrac{\text{R.P.}}{\text{W.P.}} = \eta_h \end{array} \right) \qquad \text{...(18.6)}$$

If shaft power (S.P.) is taken in kW then water power should also be taken in kW. Shaft power is commonly represented by P. But from equation (18.3*A*),

$$\text{Water power in kW} = \frac{\rho \times g \times Q \times H}{1000}, \text{ where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \qquad \eta_o = \frac{\text{Shaft power in kW}}{\text{Water power in kW}} = \frac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)} \qquad \text{...(18.6A)}$$

where P = Shaft power.

Classification of Turbine

The hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbines. Thus the following are the important classifications of the turbines :

1. According to the type of energy at inlet :
   (*a*) Impulse turbine, and (*b*) Reaction turbine.
2. According to the direction of flow through runner :
   (*a*) Tangential flow turbine, (*b*) Radial flow turbine,
   (*c*) Axial flow turbine, and (*d*) Mixed flow turbine.
3. According to the head at the inlet of turbine :
   (*a*) High head turbine, (*b*) Medium head turbine, and
   (*c*) Low head turbine.
4. According to the specific speed of the turbine :
   (*a*) Low specific speed turbine, (*b*) Medium specific speed turbine, and
   (*c*) High specific speed turbine.

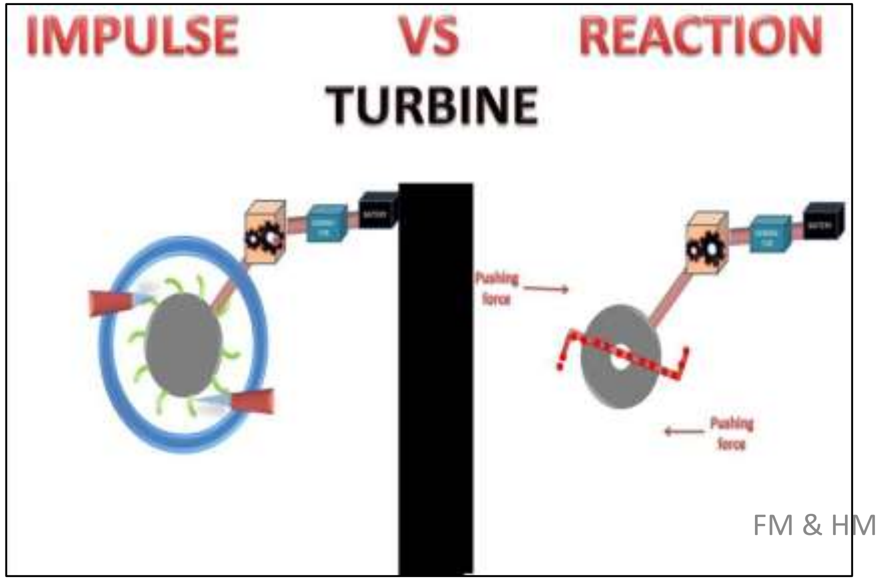
If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as **impulse turbine.** As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **reaction turbine**. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

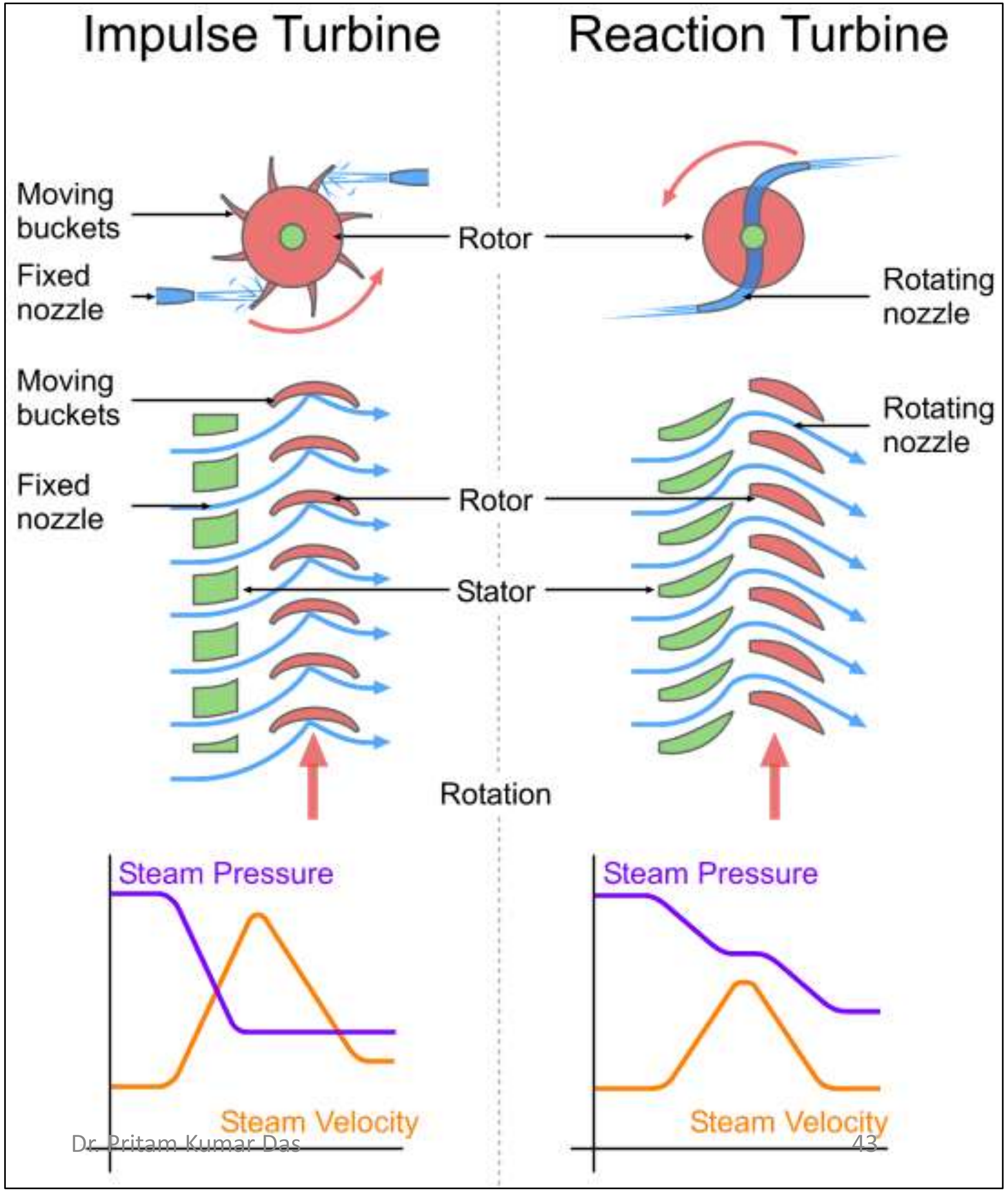
Impulse Turbine

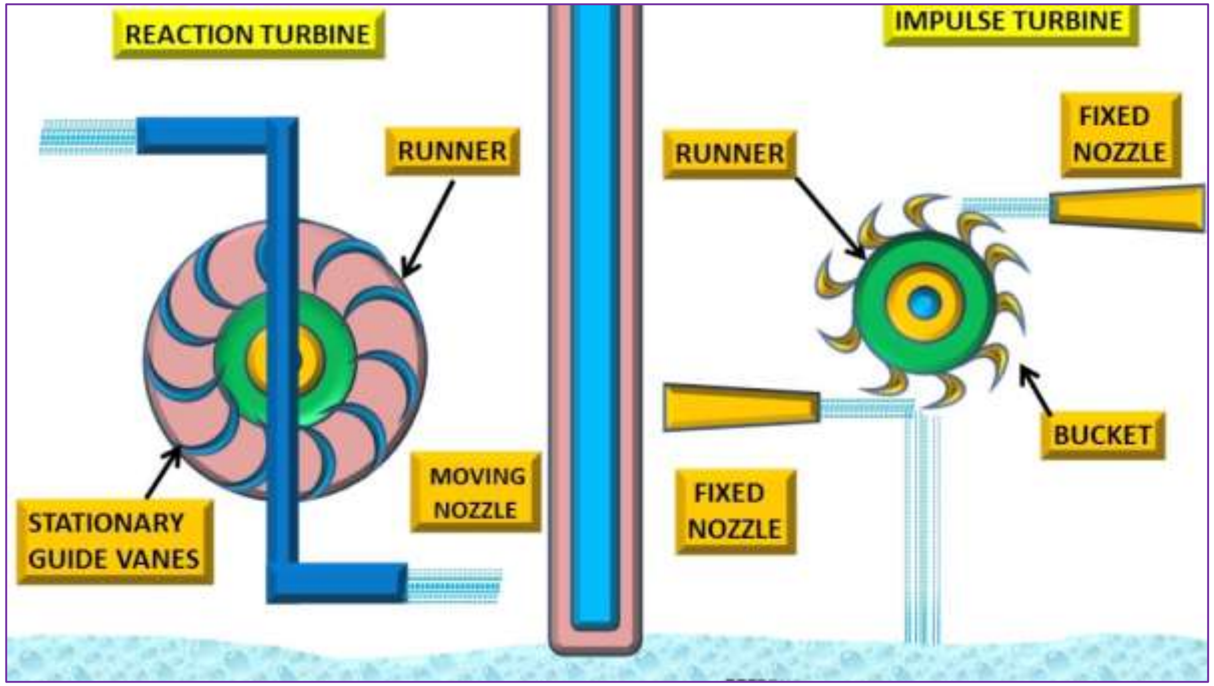
- An impulse turbine is a turbine that is driven by high velocity jets of water or steam from a nozzle directed on to vanes or buckets attached to a wheel.
- In principle, the impulse steam turbine consists of a casing containing stationary steam nozzles and a rotor with moving or rotating buckets.
- When the steam passes through the stationary nozzles and is directed at high velocity against the rotor buckets. The rotor buckets starts to rotate at high speed.

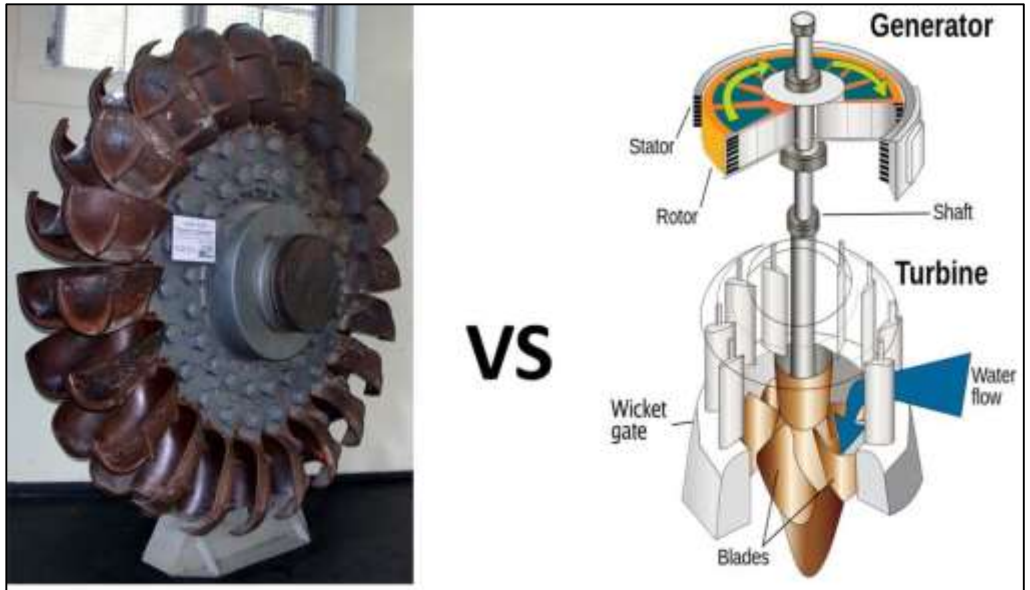
If the water flows along the tangent of the runner, the turbine is known as **tangential flow turbine.** If the water flows in the radial direction through the runner, the turbine is called **radial flow turbine**. If the water flows from outwards to inwards, radially, the turbine is known as **inward** radial flow turbine, on the other hand, if water flows radially from inwards to outwards, the turbine is known as **outward** radial flow turbine. If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine. If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called **mixed flow** turbine.

Impulse turbine	*Reaction turbine*
➢ The steam completely expands in the nozzle and its pressure remains constant during its flow through the blade passages	➢ The steam expands partially in the nozzle and further expansion takes place in the rotor blades
➢ The relative velocity of steam passing over the blade remains constant in the absence of friction	➢ The relative velocity of steam passing over the blade increases as the steam expands while passing over the blade
➢ Blades are symmetrical	➢ Blades are asymmetrical
➢ The pressure on both ends of the moving blade is same	➢ The pressure on both ends of the moving blade is different
➢ For the same power developed, as pressure drop is more, the number of stages required are less	➢ For the same power developed, as pressure drop is small, the number of stages required are more
➢ The blade efficiency curve is less flat	➢ The blade efficiency curve is more flat
➢ The steam velocity is very high and therefore the speed of turbine is high.	➢ The steam velocity is not very high and therefore the speed of turbine is low.









Difference between Impulse and Reaction Turbine

1. In impulse turbine only kinetic energy is used to rotate the turbine.	1. In reaction turbine both kinetic and pressure energy is used to rotate the turbine.
2. In this turbine water flow through the nozzle and strike the blades of turbine.	2. In this turbine water is guided by the guide blades to flow over the turbine.
3. All pressure energy of water converted into kinetic energy before striking the vanes.	3. In reaction turbine, there is no change in pressure energy of water before striking.
4. The pressure of the water remains unchanged and is equal to atmospheric pressure during process.	4. The pressure of water is reducing after passing through vanes.

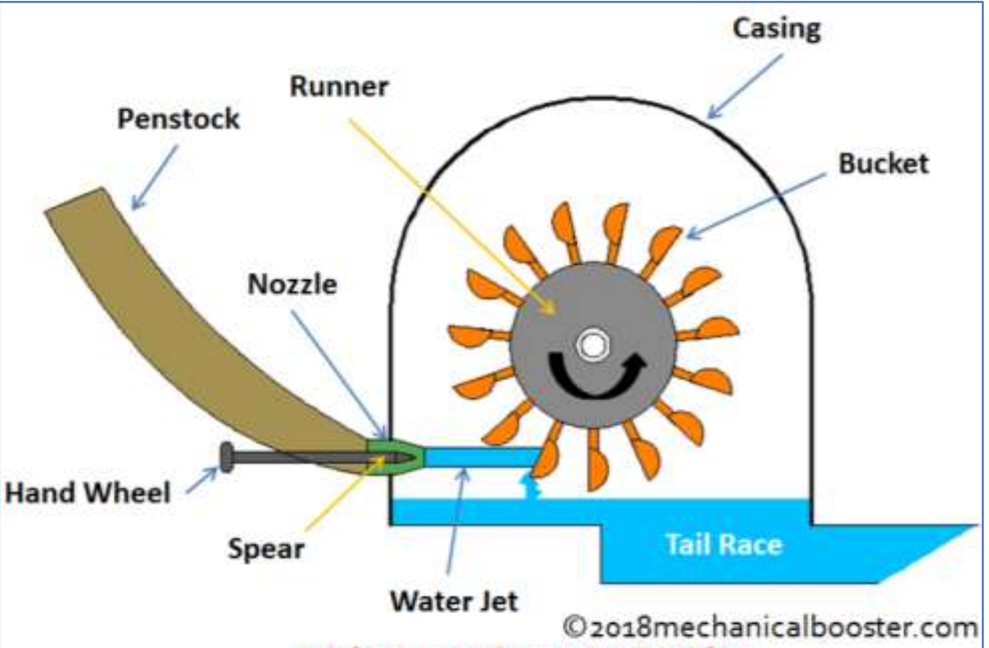
PELTON WHEEL

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

Fig. 18.1 shows the layout of a hydroelectric power plant in which the turbine is Pelton wheel . The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner. The main parts of the Pelton turbine are :

1. Nozzle and flow regulating arrangement (spear),
2. Runner and buckets,
3. Casing, and
4. Breaking jet.

1. Nozzle and Flow Regulating Arrangement. The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. 18.2. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.



Main Parts of Impulse Turbine

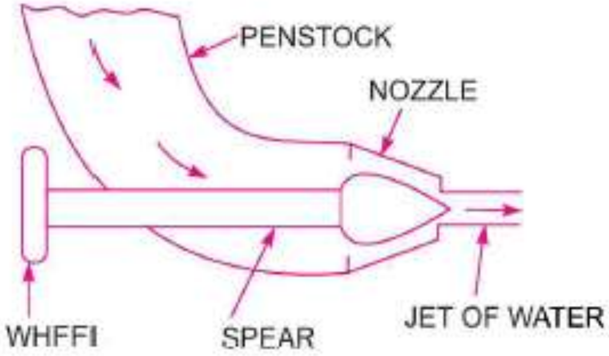


Fig. 18.2 *Nozzle with a spear to regulate flow.*

2. Runner with Buckets. Fig. 18.3 shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

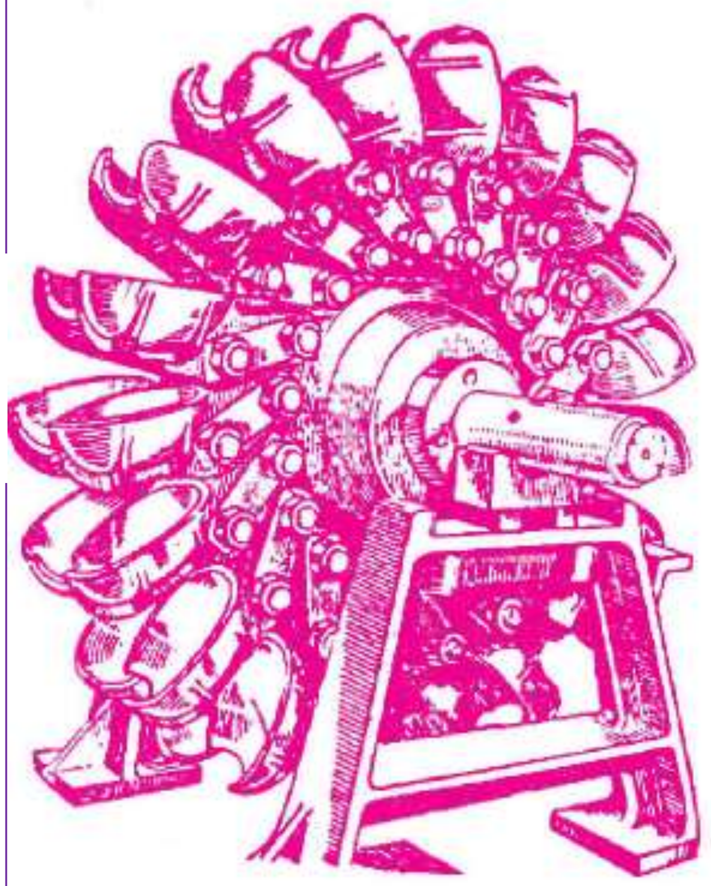


Fig. 18.3 *Runner of a pelton wheel.*

3. Casing. Fig. 18.4 shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. Breaking Jet. When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

18.6.1 Velocity Triangles and Work done for Pelton Wheel. Fig. 18.5 shows the shape of the vanes or buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. Fig. 18.5 (*b*) shows the section of the bucket at *Z-Z*. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at the outer edge of the bucket, by the same method as explained in Chapter 17.

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \frac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet $= \sqrt{2gH}$ ...(18.7)

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0^\circ \text{ and } \theta = 0^\circ$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] \quad ...(18.8)$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (18.8), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 [V_{w_1} + V_{w_2}] \times u \text{ Nm/s} \quad ...(18.9)$$

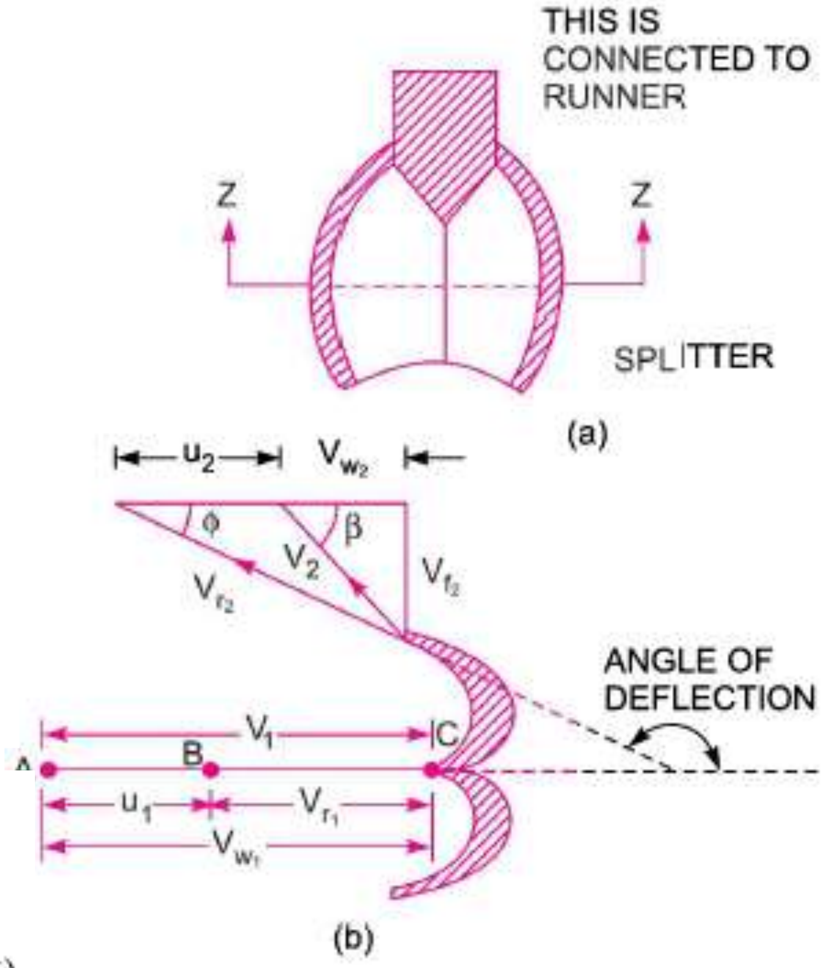


Fig. 18.5 *Shape of bucket.*

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW} \qquad ...(18.10)$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \qquad ...(18.11)$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2} m V^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \frac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \qquad ...(18.12)$$

Now $\quad V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore \quad V_{r_2} = (V_1 - u)$

and $\quad V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (18.12),

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\,u}{V_1^2}. \qquad ...(18.13)$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \quad \frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \quad \text{or} \quad \frac{d}{du}[2uV_1 - 2u^2] = 0 \quad \left(\because \frac{1+\cos\phi}{V_1^2} \neq 0\right)$$

$$\text{or} \quad 2V_1 - 4u = 0 \quad \text{or} \quad u = \frac{V_1}{2} \quad \text{...(18.14)}$$

Equation (18.14) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in equation (18.13).

$$\therefore \quad \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1+\cos\phi)}{2}. \quad \text{...(18.1}$$

18.6.2 Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \text{ (= 12 for most cases)} \quad \text{...(18.16)}$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5 \text{ m} \quad \text{...(18.17)}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Problem 18.1 *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7\ m^3/s$, Head of water, $H = 30$ m

Angle of deflection $= 160°$

∴ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

$$V_{r_1} = V_1 - u_1 = 23.77 - 10$$
$$= 13.77 \text{ m/s}$$
$$V_{w_1} = V_1 = 23.77 \text{ m/s}$$

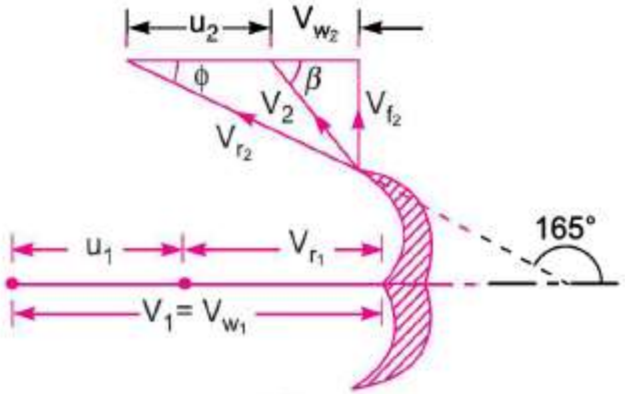


Fig. 18.6

From outlet velocity triangle,

$$V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$$
$$V_{w_2} = V_{r_2}\cos\phi - u_2$$
$$= 13.77\cos 20° - 10.0 = 2.94 \text{ m/s}$$

Work done by the jet per second on the runner is given by equation (18.9) as

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$
$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10 \qquad (\because \quad aV_1 = Q = 0.7\ m^3/s)$$
$$= 186970 \text{ Nm/s}$$

∴ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$= 0.9454$ or **94.54%. Ans.**

Problem 18.2 *A Pelton wheel is to be designed for the following specifications :*
Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :

(i) The wheel diameter, *(ii) The number of jets required, and*
(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ and $K_{u_1} = 0.45$

Shaft power, S.P. = 11,772 kW
Head , $H = 380$ m
Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85$ m/s

But $u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$

But $\frac{d}{D} = \frac{1}{6}$

$\therefore$ Dia. of jet, $d = \frac{1}{6} \times D = \frac{0.989}{6} = $ **0.165 m. Ans.**

Discharge of one jet, q = Area of jet × Velocity of jet

$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3/\text{s} = 1.818 \text{ m}^3/\text{s}$

Now $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$

$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}$, where Q = Total discharge

$\therefore$ Total discharge, $Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3/\text{s}$

$\therefore$ Number of jets $= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} =$ **2 jets. Ans.**

Problem 18.3 *The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m³/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and C_v = 1.0.*

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0$ m³/s

Angle of deflection = 165°

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio = 0.45

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

or

Gross head, $H_g = 500$ m

Head lost in friction, $h_f = \dfrac{H_g}{3} = \dfrac{500}{3} = 166.7$ m

$$\therefore \quad V_{r_1} = V_1 - u_1 = 80.86 - 36.387$$
$$= 44.473 \text{ m/s}$$

Also $$V_{w_1} = V_1 = 80.86 \text{ m/s}$$

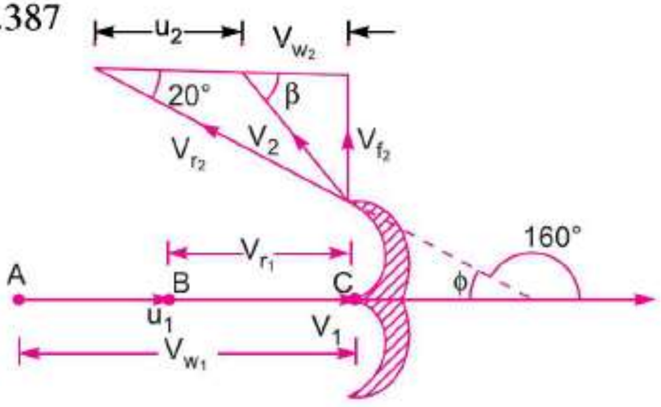


Fig. 18.7

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2}$$

$$44.473 \cos 15° = 36.387 + V_{w_2}$$

$$V_{w_2} = 44.473 \cos 15° - 36.387 = 6.57 \text{ m/s.}$$

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u \qquad (\because \quad aV_1 = Q)$$
$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$

∴ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \mathbf{6362.63\ kW.\ Ans.}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$$= \mathbf{0.9731} \text{ or } \mathbf{97.31\%.\ Ans.}$$

18.6.3 Design of Pelton Wheel. Design of Pelton wheel means the following data is to be determined :

1. Diameter of the jet (d),
2. Diameter of wheel (D),
3. Width of the buckets which is $= 5 \times d$,
4. Depth of the buckets which is $= 1.2 \times d$, and
5. Number of buckets on the wheel.

Size of buckets means the width and depth of the buckets.

Problem 18.11 *A Pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.*

Head, $H = 60$ m

Speed $N = 200$ r.p.m

Shaft power, S.P. = 95.6475 kW

Velocity of bucket, $u = 0.45 \times$ Velocity of jet

Overall efficiency, $\eta_o = 0.85$

Co-efficient of velocity, $C_v = 0.98$

Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel.

(*i*) *Velocity of jet*, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \dfrac{\pi DN}{60}$, where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60} \quad \text{or} \quad D = \frac{60 \times 15.13}{\pi \times 200} = \mathbf{1.44\ m.\ Ans.}$$

(*ii*) *Diameter of the jet* (d)

Overall efficiency $\eta_o = 0.85$

But $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}} = \dfrac{95.6475}{\left(\dfrac{\text{W.P.}}{1000}\right)} = \dfrac{95.6475 \times 1000}{\rho \times g \times Q \times H}$ $\quad (\because \ \text{W.P.} = \rho g Q H)$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$$\therefore \quad Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3/\text{s}.$$

But the discharge, Q = Area of jet $\times$ Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = \mathbf{85\ mm.\ Ans.}$$

(*iii*) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm. Ans.**

(*iv*) *Number of buckets on the wheel* is given by equation (18.17) as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 = \mathbf{23.5\ say\ 24.\ Ans.}$$

Problem 18.12 *Determine the power given by the jet of water to the runner of a Pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through the jet water is 0.03 m³/s. The side clearance angle is 15° and take $C_v = 0.975$.*

Solution. Given :

Tangential velocity of wheel, $u = u_1 = u_2 = 20$ m/s

Net head, $H = 50$ m

Discharge , $Q = 0.03\ m^3/s$

Side clearance angle, $\phi = 15°$

Co-efficient of velocity, $C_v = 0.975$

Velocity of the jet, $V_1 = C_v \times \sqrt{2gH}$

$$= 0.975 \times \sqrt{2 \times 9.81 \times 50}$$

$$= 30.54 \text{ m/s}$$

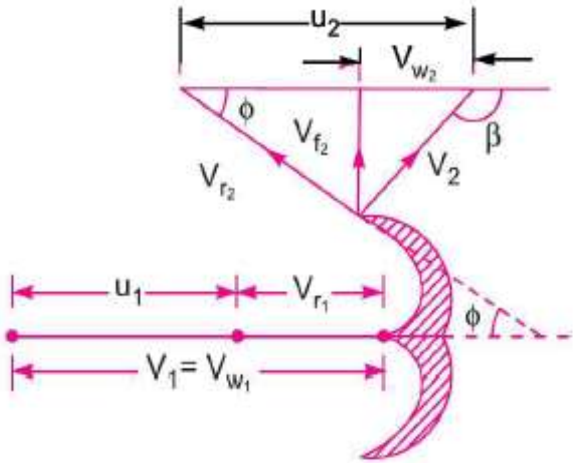


Fig. 18.9

From inlet triangle, $V_{w_1} = V_1 = 30.54$ m/s

$$V_{r_1} = V_{w_1} - u_1 = 30.54 - 20.0 = 10.54 \text{ m/s}$$

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 10.54 \text{ m/s}$$

$$V_{r_2} \cos \phi = 10.54 \cos 15° = 10.18 \text{ m/s}$$

As $V_{r_2} \cos \phi$ is less than u_2, the velocity triangle at outlet will be as shown in Fig. 18.9.

$$\therefore \quad V_{w_2} = u_2 - V_{r_2} \cos \phi = 20 - 10.18 = 9.82 \text{ m/s.}$$

Also as β is an obtuse angle, the work done per second on the runner,

$$= \rho a V_1 \left[V_{w_1} - V_{w_2}\right] \times u = \rho Q \left[V_{w_1} - V_{w_2}\right] \times u$$

$$= 1000 \times .03 \times [30.54 - 9.82] \times 20 = 12432 \text{ Nm/s}$$

Power given to the runner in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{12432}{1000} =$ **12.432 kW. Ans.**